

9.4) The product rule

Worked example

Differentiate with respect to x :

$$y = x^2 e^{3x}$$

$$f(x) = x^5 e^{2x}$$

Your turn

Differentiate with respect to x :

$$y = x^3 e^{2x}$$

$$\frac{dy}{dx} = x^2 e^{2x} (2x + 3)$$

Worked example

Differentiate with respect to x :

$$y = x^2 \ln 3x$$

$$f(x) = x^4 \ln 5x$$

Your turn

Differentiate with respect to x :

$$y = x^3 \ln 2x$$

$$\frac{dy}{dx} = x^2(1 + 3 \ln 2x)$$

Worked example

Differentiate with respect to x :

$$y = 2x^3(4x - 5)^6$$

$$f(x) = 6x^5(4x - 3)^2$$

Your turn

Differentiate with respect to x :

$$y = 3x^2(6x - 5)^4$$

$$\frac{dy}{dx} = 6x(6x - 5)^3(18x - 5)$$

Worked example

Differentiate with respect to x :

$$y = x^3 \sin x$$

$$f(x) = x^4 \cos x$$

Your turn

Differentiate with respect to x :

$$y = x^2 \sin x$$

$$\frac{dy}{dx} = x(x \cos x + 2 \sin x)$$

Worked example

Differentiate with respect to x :

$$y = e^{3x} \sin^4 2x$$

$$f(x) = e^{2x} \cos^3 4x$$

Your turn

Differentiate with respect to x :

$$y = e^{4x} \sin^2 3x$$

$$\frac{dy}{dx} = e^{4x} \sin 3x (6 \cos 3x + 4 \sin 3x)$$

Worked example

Determine the coordinates of the turning point:

$$y = xe^{3x}$$

$$f(x) = xe^{4x}$$

Your turn

Determine the coordinates of the turning point:

$$y = xe^{2x}$$

$$\left(-\frac{1}{2}, -\frac{1}{2e}\right)$$

Worked example

Differentiate with respect to x :

$$y = x^2\sqrt{5x - 2}$$

Your turn

Differentiate with respect to x :

$$y = x^2\sqrt{3x - 1}$$

$$\frac{dy}{dx} = \frac{x(15x - 4)}{2\sqrt{3x - 1}}$$

Worked example

Find the equation of the tangent to the curve with equation $y = x^2 \sin(x^2)$ at the point $\left(\frac{\sqrt{\pi}}{2}, \frac{\pi\sqrt{2}}{8}\right)$ in the form $ax + by + c = 0$ where a, b and c are exact constants.

Your turn

Find the equation of the tangent to the curve with equation $y = x^2 \cos(x^2)$ at the point $\left(\frac{\sqrt{\pi}}{2}, \frac{\pi\sqrt{2}}{8}\right)$ in the form $ax + by + c = 0$ where a, b and c are exact constants.

$$\sqrt{2\pi}(\pi - 4)x + 8y - \pi\sqrt{2}\left(\frac{\pi - 2}{2}\right) = 0$$

Worked example

Differentiate with respect to x :

$$\frac{dy}{dx} = \frac{1}{5x(x+1)^{\frac{1}{2}}}$$

Your turn

Differentiate with respect to x :

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

$$\frac{d^2y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$$