9.3) The chain rule

Worked example	Your turn
Differentiate with respect to x: $y = (4x^3 - x)^7$ $f(x) = (7x^2 - 3x)^4$	Differentiate with respect to x: $y = (3x^4 + x)^5$ $\frac{dy}{dx} = 5(3x^4 + x)^4(12x^3 + 1)$

Worked example	Your turn
Differentiate with respect to x: $y = (\ln x)^3$	Differentiate with respect to x: $y = (\ln x)^5$
	$\frac{dy}{dx} = \frac{5(\ln x)^4}{x}$
$f(x) = (\ln x)^4$	

Worked example	Your turn
Differentiate with respect to x: $y = e^{x^2+x}$ $f(x) = e^{x^3-2x+1}$	Differentiate with respect to x: $y = e^{x^4-3x^2-1}$ $\frac{dy}{dx} = (4x^3 - 6x)e^{x^4-3x^2-1}$

Worked example	Your turn
Differentiate with respect to x: $y = (3^x + 1)^2$	Differentiate with respect to x: $y = (4^x - 2x)^3$
$f(x) = (2^x - 3x)^4$	$\frac{dy}{dx} = 3(4^x - 2x)^2(4^x \ln 4 - 2)$

Worked example	Your turn
Differentiate with respect to x: $$y = \ln(\sin x)$$	Differentiate with respect to x: $$y = \ln(\cos x)$$ $$\frac{dy}{dx} = \tan x$$

Worked example	Your turn
Differentiate with respect to x: $y = \ln(x^2 - 3x + 4)$	Differentiate with respect to x: $y = \ln(x^4 - 5x^2 - 3)$
$f(x) = \ln(x^3 - 2x - 5)$	$\frac{dy}{dx} = \frac{4x^3 - 10x}{x^4 - 5x^2 - 3}$

Worked example	Your turn
Differentiate with respect to x: $y = \sin 5x$ $f(x) = \cos(-4x)$	Differentiate with respect to x: $y = \cos 3x$ $\frac{dy}{dx} = -3\sin 3x$

Worked example	Your turn
Differentiate with respect to x: $y = \sin^4 x$ $f(x) = \cos^3 x$	Differentiate with respect to x: $y = \sin^2 x$ $\frac{dy}{dx} = 2\sin x \cos x = \sin 2x$

Worked example	Your turn
Differentiate with respect to x: $y = \sin^4 3x$ $f(x) = \cos^3 2x$	Differentiate with respect to x: $y = \sin^2 3x$ $\frac{dy}{dx} = 6\sin 3x \cos 3x = 3\sin 6x$

Worked example	Your turn
Differentiate with respect to x: $y = \sqrt{2x - 1}$ $f(x) = \sqrt[3]{4x^2 + 5}$	Differentiate with respect to x: $y = \sqrt[4]{2x^3 + 1}$ $\frac{dy}{dx} = \frac{1}{4}(2x^3 + 1)^{-\frac{3}{4}}(6x^2)$

Worked example	Your turn
Differentiate with respect to x: $$y = e^{e^{-x}}$$	Differentiate with respect to x: $$y = e^{e^{x}}$$ $$\frac{dy}{dx} = e^{x}\left(e^{e^{x}}\right)$$

Worked example	Your turn
Given that $y = \sqrt{2x^5 - 2}$, find $\frac{dy}{dx}$ at $(3,22)$	Given that $y = \sqrt{5x^2 + 1}$, find $\frac{dy}{dx}$ at $(4,9)$ $\frac{20}{9}$

Worked example	Your turn
A curve C has equation $$y = \frac{3}{(2-5x)^4}, x \neq \frac{2}{5}$$ Find an equation for the normal to C at the point with x-coordinate 1 in the form $ax + by + c = 0$, where a, b and c are integers	A curve C has equation $$y = \frac{4}{(3-2x)^5}, x \neq \frac{3}{2}$$ Find an equation for the normal to C at the point with x-coordinate 2 in the form $ax + by + c = 0$, where a, b and c are integers $$x + 40y + 158 = 0$$