9.1) Differentiating $\sin x$ and $\cos x$

Worked example	Your turn
Prove, from first principles, that the derivative of $\sin x$ is $\cos x$. You may assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$	Prove, from first principles, that the derivative of $\sin x$ is $\cos x$. You may assume that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

Let $f(x) = \sin x$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0} \frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h} \\
&= \lim_{h\to 0} \left(\left(\frac{\cos h - 1}{h} \right) \sin x + \left(\frac{\sin h}{h} \right) \cos x \right) \\
&= \lim_{h\to 0} ((0)\sin x + (1)\cos x) \\
&= \cos x
\end{aligned}
$$

Worked example	Your turn
Find: $$\frac{d}{dx}(\sin 5x)$$	Find: $$\frac{d}{dx}(\sin \frac{1}{2}x)$$ $$\frac{1}{2}\cos 2x$$
$$\frac{d}{dx}(\cos \frac{1}{2}x)$$	$$\frac{d}{dx}(\cos 5x)$$ $$-5\sin 5x$$
$$\frac{d}{dx}(6\sin 7x - 4\cos 3x)$$	$$\frac{d}{dx}(7\sin 6x - 3\cos 4x)$$ $$42\cos 6x + 12\sin 4x$$

Worked example	Your turn
A curve has equation $y = \frac{1}{4}x - \cos 3x$. Find the stationary points on the curve in the interval $0 \leq x \leq \pi$	A curve has equation $y = \frac{1}{2}x - \cos 2x$. Find the stationary points on the curve in the interval $0 \leq x \leq \pi$ $(1.70, 1.82)$ and $(3.02, 0.539)$ (3 sf)

Worked example	Your turn
A curve has equation $y = \sin 5x + 3x$. Find the stationary points on the curve in the interval $0 \leq x \leq \frac{3}{5}\pi$	A curve has equation $y = \sin 3x + 2x$. Find the stationary points on the curve in the interval $0 \leq x \leq \frac{2}{3}\pi$ $(0.767, 2.279)$ and $(1.328, 1.910)$ (3 sf)

Worked example	Your turn
A curve has equation $y = \sin 4x - \cos 3x$. Find the equation of the tangent to the curve at the point $(\pi, 1)$	A curve has equation $y = \sin 4x + \cos 3x$. Find the equation of the tangent to the curve at the point $(\pi, -1)$ $y = 4x - 4\pi - 1$

Worked example	Your turn
A curve has equation $y = 3x^2 - \sin x$. Show that the equation of the normal to the curve at the point with x-coordinate π is $x + (6\pi + 1)y - \pi(18\pi^2 - 3\pi - 1) = 0$	A curve has equation $y = 3x^2 + \sin x$. Show that the equation of the normal to the curve at the point with x-coordinate π is $x + (6\pi - 1)y - \pi(18\pi^2 - 3\pi + 1) = 0$ Shown