

Standard Derivatives

y	$\frac{dy}{dx}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\operatorname{cosec}(x)$	$-\operatorname{cosec}(x)\cot(x)$
$\cot(x)$	$-\operatorname{cosec}^2(x)$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
a^x	$a^x \ln(a)$

9C (Part 2) The Chain Rule

1. Given that $y = (3x^4 + x)^5$, find $\frac{dy}{dx}$ using the chain rule.

2. Given that $y = \sin^4 x$, find $\frac{dy}{dx}$

3. Given that $y = \sqrt{5x^2 + 1}$, find the gradient of the curve at (4,9).

9D The Product Rule

1. Given that $f(x) = x^2\sqrt{3x-1}$, find $f'(x)$.

2. Given that $y = e^{4x} \sin^2 3x$, show that $\frac{dy}{dx} = e^{4x} \sin 3x (A \cos 3x + B \sin 3x)$, where A and B are constants to be found.

9E The Quotient Rule

1. Given that $y = \frac{x}{2x+5}$, find $\frac{dy}{dx}$

2. A curve C has equation $y = \frac{\sin x}{e^{2x}}$, $0 < x < \pi$.

The curve has a stationary point at P . Find the coordinates of P , to 3 significant figures.

9G Differentiating Parametric Equations

1. Find the gradient at the point P where $t = 2$, on the curve given parametrically by:

$$x = t^3 + t, \quad y = t^2 + 1, \quad t \in \mathbb{R}$$

2. Find the equation of the normal at the point P , where $\theta = \frac{\pi}{6}$, to the curve with parametric equations $x = 3\sin\theta$ and $y = 5\cos\theta$.

9C (Part 1) dx/dy

1. Find the value of $\frac{dy}{dx}$ at the point (2,1) on the curve with equation $y^3 + y = x$

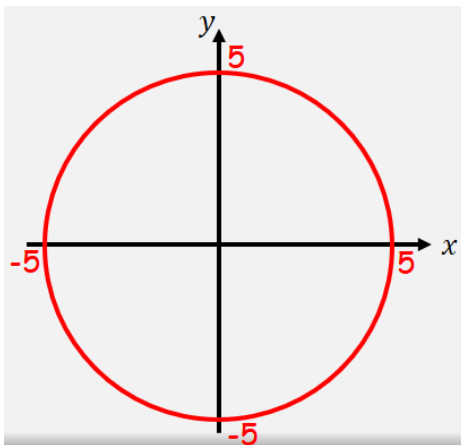
9H Implicit Differentiation

1. Differentiate the following equation implicitly:

$$y^3 = 3x^2$$

2. Below is a sketch of the circle with equation $x^2 + y^2 = 25$, $-5 \leq x \leq 5$, $-5 \leq y \leq 5$.

Find the gradient of the curve where $x = 4$



3. Find $\frac{dy}{dx}$ in terms of x and y when:

$$x^3 + x + y^3 + 3y = 6$$

4. Given that $4xy^2 + \frac{6x^2}{y} = 10$, find the value of $\frac{dy}{dx}$ at the point (1,1)

5. Find the value of $\frac{dy}{dx}$ at the point (1,1), when:

$$e^{2x} \ln y = x + y + 2$$

Proving Derivatives

Small angle notes

Differentiating Sine from First Principles

Differentiating $\sin(kx)$ from First Principles

Differentiating Cosine from First Principles

Differentiating Sec with the Chain Rule

Differentiating Cosec with the Chain Rule

Differentiating Tan with the Quotient Rule

Differentiating Cot with the Quotient Rule

Differentiating $\ln(x)$ using implicit differentiation

Differentiating a^x using logs and implicit differentiation

9I Concave & Convex Functions

1. Find the interval on which the function $f(x) = x^3 + 4x + 3$ is concave.

2. Show that the function $f(x) = e^{2x} + x^2$ is convex for all real values of x .

3. The curve C has equation $y = x^3 - 2x^2 - 4x + 5$

a) Show that C is concave on the interval $[-2,0]$ and convex on the interval $[1,3]$

b) Find the coordinates of the point of inflection

9J Chain Rule in Context

1. Given that the area of a circle $A \text{ cm}^2$ is related to its radius $r \text{ cm}$ by the formula $A = \pi r^2$, and that the rate of change of its radius in cm s^{-1} is given by $\frac{dr}{dt} = 5$, find $\frac{dA}{dt}$ when $r = 3$

2. The volume of a hemisphere $V \text{ cm}^3$ is related to its radius $r \text{ cm}$ by the formula $V = \frac{2}{3}\pi r^3$, and the total surface area $S \text{ cm}^2$ is given by the formula $S = 3\pi r^2$. Given that the rate of increase of volume, in $\text{cm}^3 \text{ s}^{-1}$, $\frac{dV}{dt} = 6$, find the rate of increase of the surface area, $\frac{dS}{dt}$.

3. In the decay of radioactive particles, the rate at which particles decay is proportional to the number of particles remaining. Write down a differential equation for the rate of change of the number of particles.

4. Newton's law of cooling states that the rate of loss of temperature of a body is proportional to the excess temperature of the body compared to its surroundings. Write an equation that expresses this law.

5. The head of a snowman of radius R cm loses volume by evaporation at a rate proportional to its surface area. Assuming that the head is spherical, that the volume of a sphere is given by $V = \frac{4}{3}\pi R^3$ cm³ and that the surface area is $A = 4\pi R^2$ cm², write down a differential equation for the rate of change of radius of the snowman's head.