9A Differentiating Sine & Cosine

Small angle notes

Differentiating Sine from First Principles

Differentiating Sin(kx) from First Principles

1. Find $\frac{dy}{dx}$ given that:

a)

$$y = sin2x$$

b)

$$y = 3sin2x$$

c)

$$y = 3cosx + 2sin4x$$

2. A curve has equation:

$$y = \frac{1}{2}x - cos2x$$

Find the stationary points on the curve in the interval $0 \leq x \leq \pi$.

9B Differentiating Exponentials

e^x

$\ln(x)$

a^x

1. Find $\frac{dy}{dx}$ given that:

$$y = e^{3x} + 2^{3x}$$

2. Find $\frac{dy}{dx}$ given that:

$$y = ln(x^3) + ln7x$$

3. Find $\frac{dy}{dx}$ given that:

$$y = \frac{2 - 3e^{7x}}{4e^{3x}}$$

9C (Part 1) dx/dy

1. Find the value of $\frac{dy}{dx}$ at the point $(2,1)$ on the curve with equation $y^3 + y = x$

9C (Part 2) The Chain Rule

1. Given that $y = (3x^4 + x)^5$, find $\frac{dy}{dx}$ using the chain rule.

2. Given that $y = sin^4 x$, find $\frac{dy}{dx}$

3. Given that $y = \sqrt{5x^2 + 1}$, find the gradient of the curve at $(4,9)$.

9D The Product Rule

1. Given that $f(x) = x^2\sqrt{3x-1}$, find $f'(x)$.

2. Given that $y = e^{4x} sin^2 3x$, show that $\frac{dy}{dx} = e^{4x} sin3x(Acos3x + Bsin3x)$, where A and B are constants to be found.

9E The Quotient Rule

1. Given that $y = \frac{x}{2x+5}$, find $\frac{dy}{dx}$

2. A curve C has equation $y = \frac{sinx}{e^{2x}}, \quad 0 < x < \pi.$

The curve has a stationary point at P. Find the coordinates of P, to 3 significant figures.

9F Trigonometric Derivatives

1. Given that $y = tanx$, find $\frac{dy}{dx}$

2. Given that $y = cosecx$, find $\frac{dy}{dx}$.

3. Given that $y = secx$, find $\frac{dy}{dx}$.

4. Given that $y = cotx$, find $\frac{dy}{dx}$.

5. Differentiate:

$$y = xtan2x$$

6. Differentiate:

$$y = tan^4x$$

7. Differentiate:

$$y = \frac{cosec 2x}{x^2}$$

8. Differentiate:

$$y = sec^3 x$$

No Arc derivatives on new Spec

9G Differentiating Parametric Equations

1. Find the gradient at the point P where $t = 2$, on the curve given parametrically by:

$$x = t^3 + t,\ y = t^2 + 1,\ t \in \mathbb{R}$$

2. Find the equation of the normal at the point P, where $\theta = \frac{\pi}{6}$, to the curve with parametric equations $x = 3sin\theta$ and $y = 5cos\theta$.

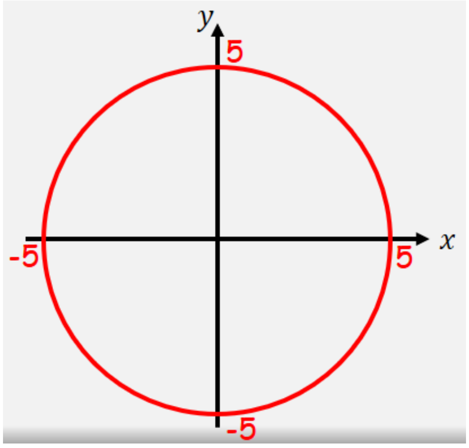
9H Implicit Differentiation

1. Differentiate the following equation implicitly:

$$y^3 = 3x^2$$

2. Below is a sketch of the circle with equation $x^2 + y^2 = 25, -5 \leq x \leq 5, -5 \leq y \leq 5$.

Find the gradient of the curve where $x = 4$



3. Find $\frac{dy}{dx}$ in terms of x and y when:

$$x^3 + x + y^3 + 3y = 6$$

4. Given that $4xy^2 + \frac{6x^2}{y} = 10$, find the value of $\frac{dy}{dx}$ at the point (1,1)

5. Find the value of $\frac{dy}{dx}$ at the point (1,1), when:

$$e^{2x} lny = x + y + 2$$

9I Concave & Convex Functions

1. Find the interval on which the function $f(x) = x^3 + 4x + 3$ is concave.

2. Show that the function $f(x) = e^{2x} + x^2$ is convex for all real values of x.

3. The curve C has equation $y = x^3 - 2x^2 - 4x + 5$

a) Show that C is concave on the interval $[-2,0]$ and convex on the interval $[1,3]$

b) Find the coordinates of the point of inflection

9J Chain Rule in Context

1. Given that the area of a circle $A\ cm^2$ is related to its radius $r\ cm$ by the formula $A = \pi r^2$, and that the rate of change of its radius in $cm\ s^{-1}$ is given by $\frac{dr}{dt} = 5$, find $\frac{dA}{dt}$ when $r = 3$

2. The volume of a hemisphere $V\ cm^3$ is related to its radius $r\ cm$ by the formula $V = \frac{2}{3}\pi r^3$, and the total surface area $S\ cm^2$ is given by the formula $S = 3\pi r^2$. Given that the rate of increase of volume, in $cm^3\ s^{-1}$, $\frac{dV}{dt} = 6$, find the rate of increase of the surface area, $\frac{dS}{dt}$.

3. In the decay of radioactive particles, the rate at which particles decay is proportional to the number of particles remaining. Write down a differential equation for the rate of change of the number of particles.

4. Newton's law of cooling states that the rate of loss of temperature of a body is proportional to the excess temperature of the body compared to its surroundings. Write an equation that expresses this law.

5. The head of a snowman of radius $R\ cm$ loses volume by evaporation at a rate proportional to its surface area. Assuming that the head is spherical, that the volume of a sphere is given by $V = \frac{4}{3}\pi R^3\ cm^3$ and that the surface area is $A = 4\pi R^2\ cm^2$, write down a differential equation for the rate of change of radius of the snowman's head.