9) Trigonometric ratios

9.1) The cosine rule

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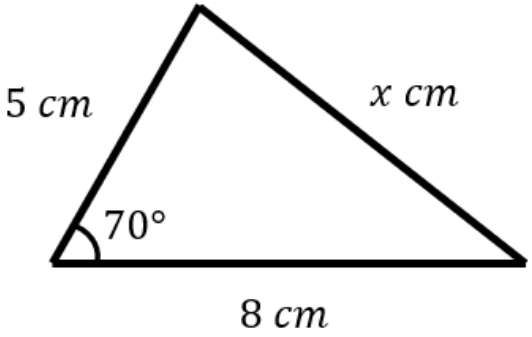
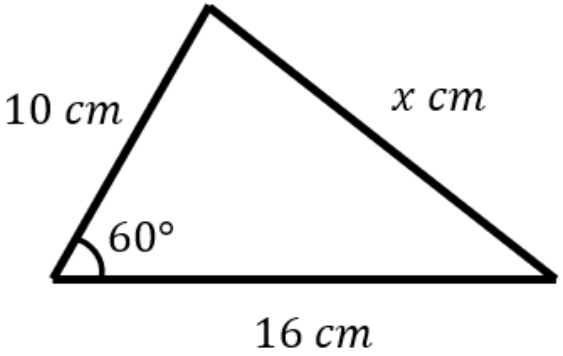
Worked example

Find the value of x



Your turn

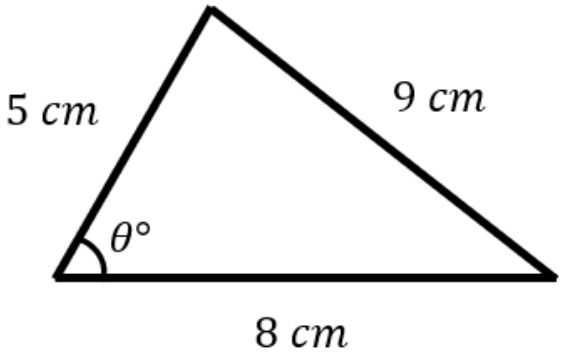
Find the value of x



$x = 14$

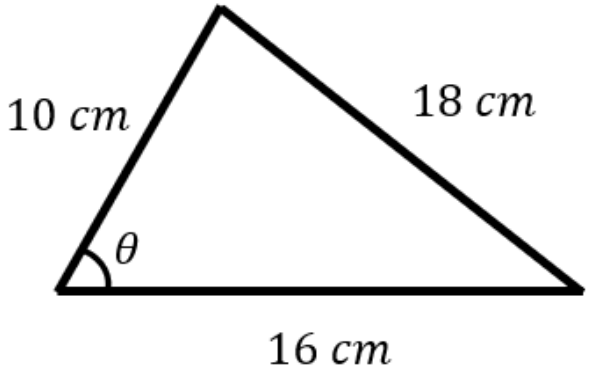
Worked example

Find the value of θ



Your turn

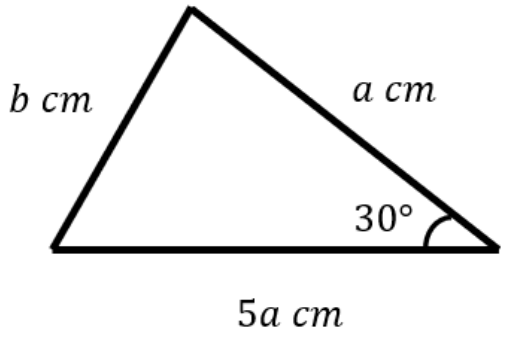
Find the value of θ



$\theta = 84.26°$ (2 dp)

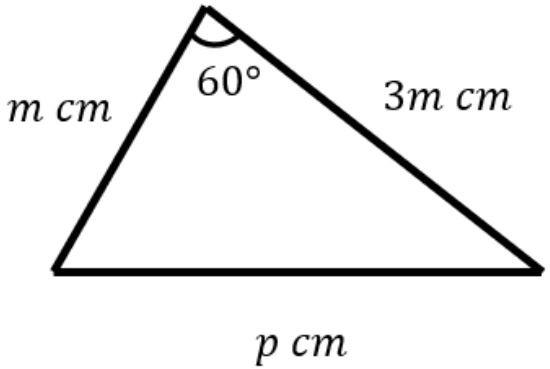
Worked example

Express b in terms of a



Your turn

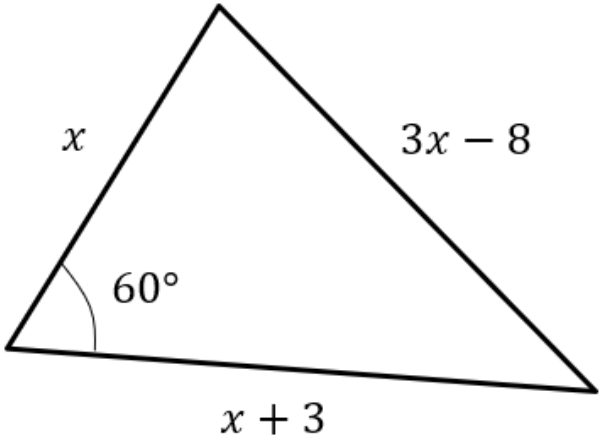
Express p in terms of m



$p = m\sqrt{7}$

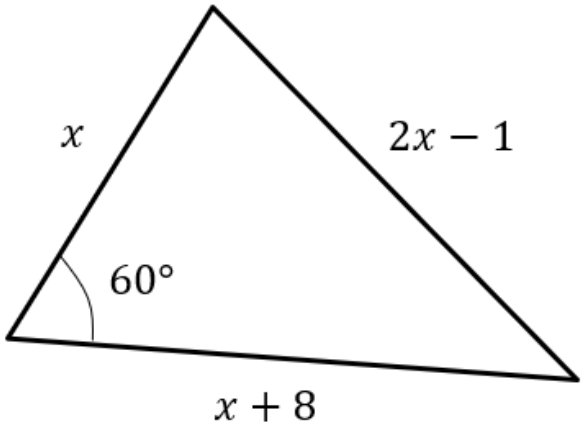
Worked example

Determine the value of x



Your turn

Determine the value of x



$x = 7$

Worked example

Find the size of the smallest angle in a triangle whose sides have lengths $6\ cm, 10cm$ and $12\ cm$

Your turn

Find the size of the smallest angle in a triangle whose sides have lengths $3\ cm, 5cm$ and $6\ cm$

$29.9°$ (3 sf)

Worked example

Coastguard station B is 16 km, on a bearing of $030°$, from coastguard station A.
A ship C is 8.4 km on a bearing of $081°$, away from A.
Calculate how far C is from B.

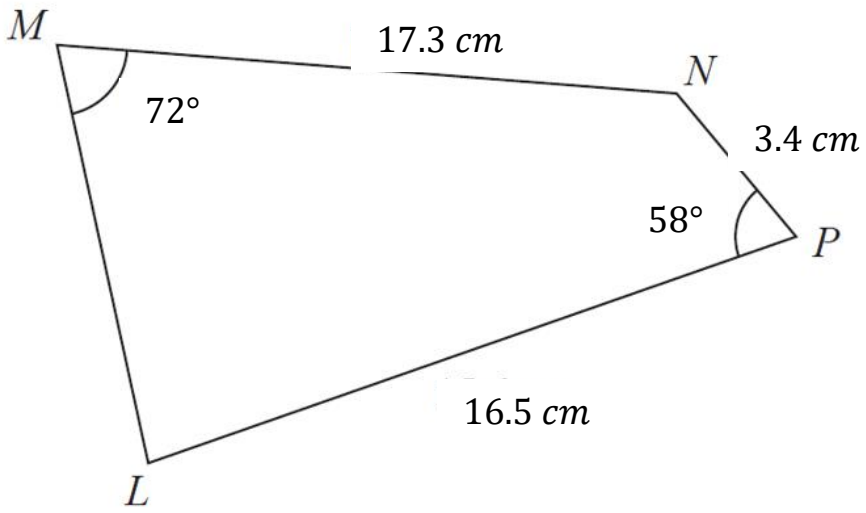
Your turn

Coastguard station B is 8 km, on a bearing of $060°$, from coastguard station A.
A ship C is 4.8 km on a bearing of $018°$, away from A.
Calculate how far C is from B.

$5.47\ km$ (3 sf)

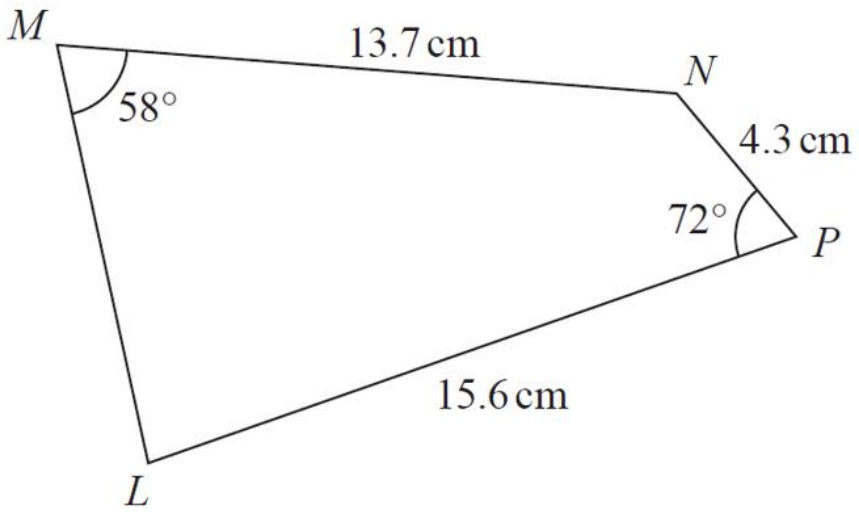
Worked example

Calculate the size of angle MLP



Your turn

Calculate the size of angle MLP



67.49° (2 dp)

9.2) The sine rule

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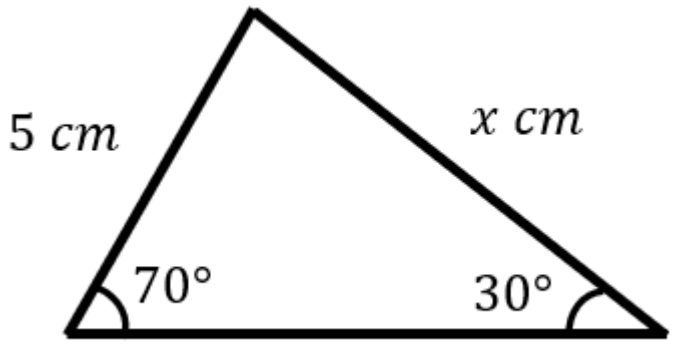
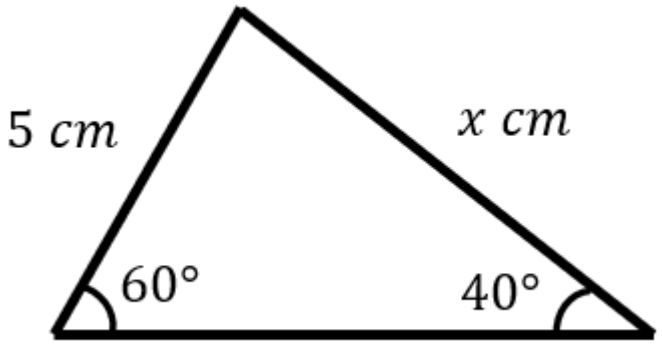
Worked example

Find the value of x



Your turn

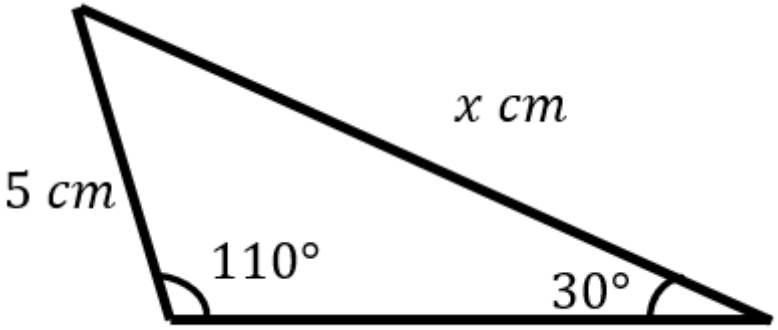
Find the value of x



$x = 6.74$ (2 dp)

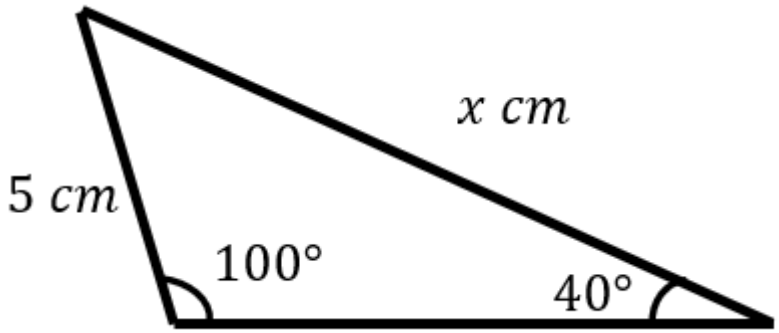
Worked example

Find the value of x



Your turn

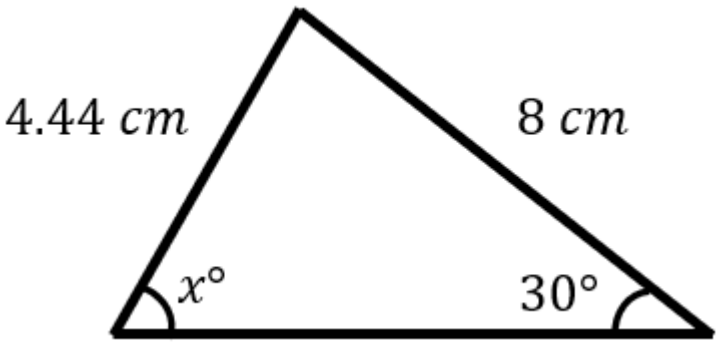
Find the value of x



$x = 7.66$ (2 dp)

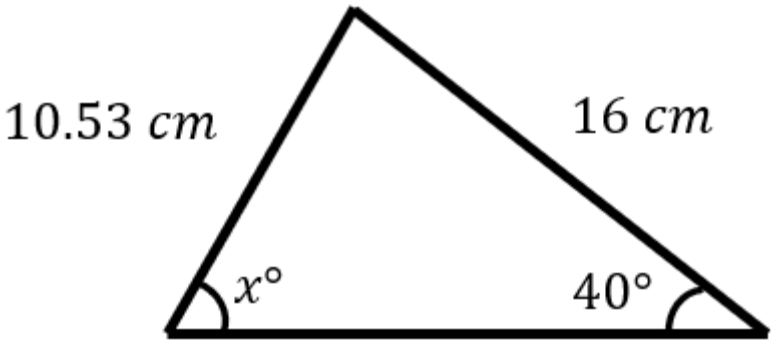
Worked example

Find x, where $x < 90$



Your turn

Find x, where $x < 90$



$x = 77.61$

Worked example

In $\triangle ABC$, $AB = 8cm$, $AC = 6\ cm$ and $\angle ABC = 88°$.

Work out the two possible values of $\angle ACB$

Your turn

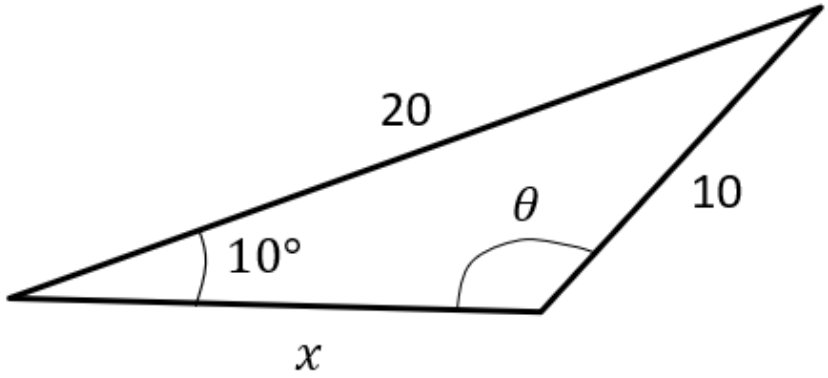
In $\triangle ABC$, $AB = 4cm$, $AC = 3\ cm$ and $\angle ABC = 44°$.

Work out the two possible values of $\angle ACB$

67.9° and 112° (3 sf)

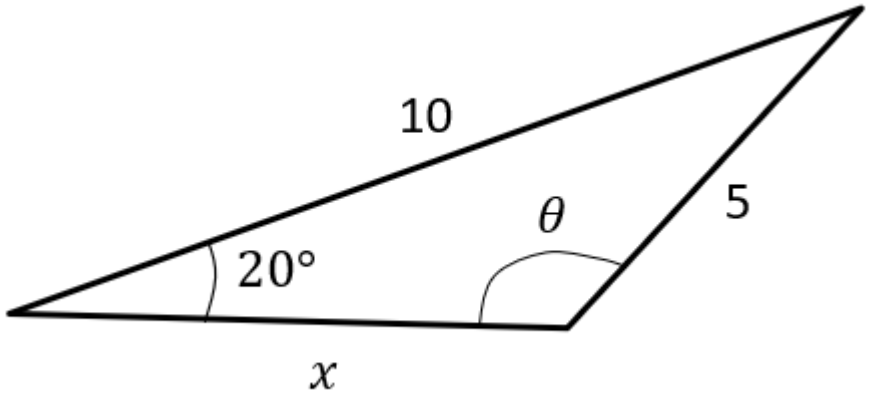
Worked example

Given that the angle θ is obtuse, determine θ and hence determine the length of x.



Your turn

Given that the angle θ is obtuse, determine θ and hence determine the length of x.



$\theta = 137°$ (3 sf)

$x = 5.75$ (3 sf)

9.3) Areas of triangles

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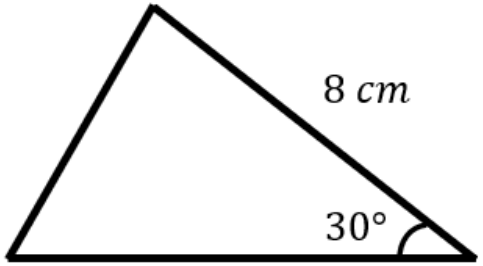
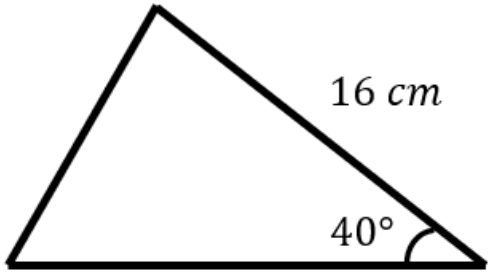
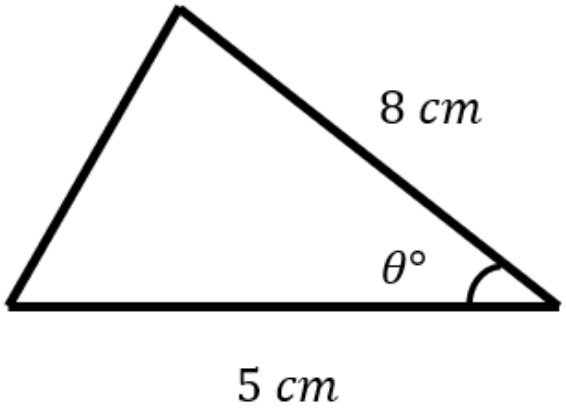
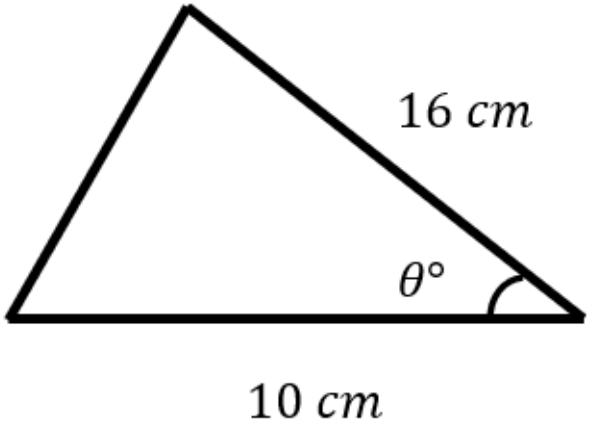
Worked example	Your turn
Calculate the area of the triangle:	Calculate the area of the triangle:



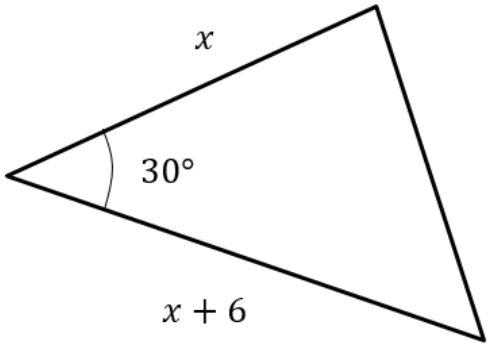


$51.42\ cm^2$ (2 dp)

Worked example	Your turn
The area is $10\ cm^2$. Angle θ is acute. Calculate θ  	The area is $51.42\ cm^2$. Angle θ is acute. Calculate θ  $\theta = 40.0$ (3 sf)

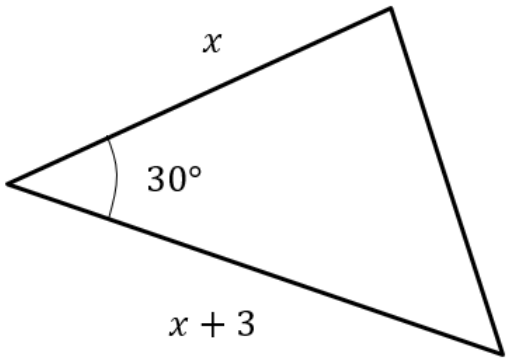
Worked example

The area is 40. Determine x



Your turn

The area is 10. Determine x



$x = 5$

Worked example	Your turn
A triangle has sides $5.1\ cm$, $3.4\ cm$ and $2.85\ cm$. Work out the area of the triangle	A triangle has sides $10.2\ cm$, $6.8\ cm$ and $5.7\ cm$. Work out the area of the triangle $18.3\ cm^2$ (1 dp)

Worked example

In $\triangle ABC, AB = 2.5cm, BC = 3\ cm$ and $\angle ABC = x$.

Given that the area of $\triangle ABC$ is $3\ cm^2$ and that AC is the longest side, find the value of x

Your turn

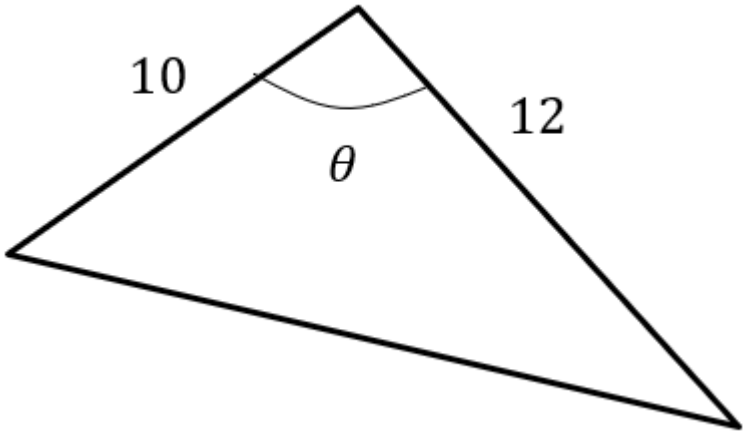
In $\triangle ABC, AB = 5cm, BC = 6\ cm$ and $\angle ABC = x$.

Given that the area of $\triangle ABC$ is $12\ cm^2$ and that AC is the longest side, find the value of x

$x = 127°$ (3 sf)

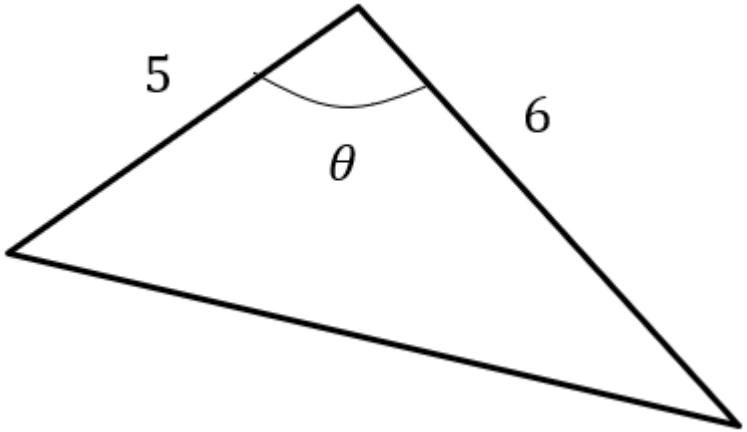
Worked example

The area of this triangle is 40.
If θ is obtuse, determine θ.



Your turn

The area of this triangle is 10.
If θ is obtuse, determine θ.



$\theta = 138°$ (3 sf)

9.4) Solving triangle problems

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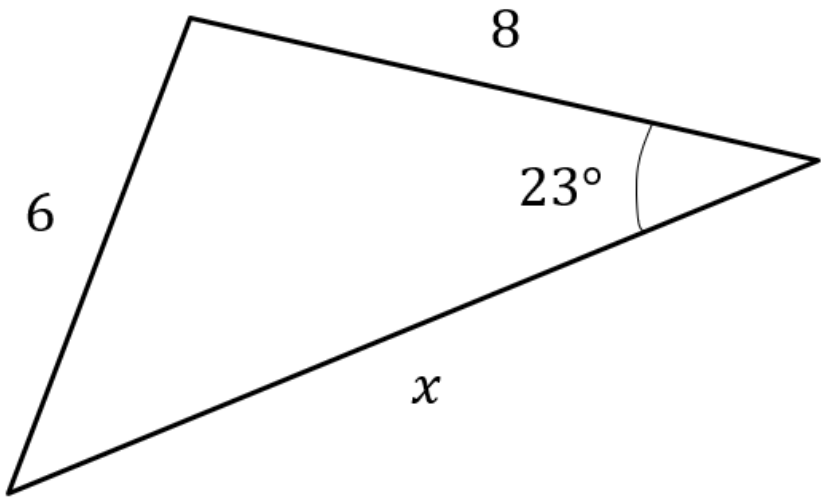
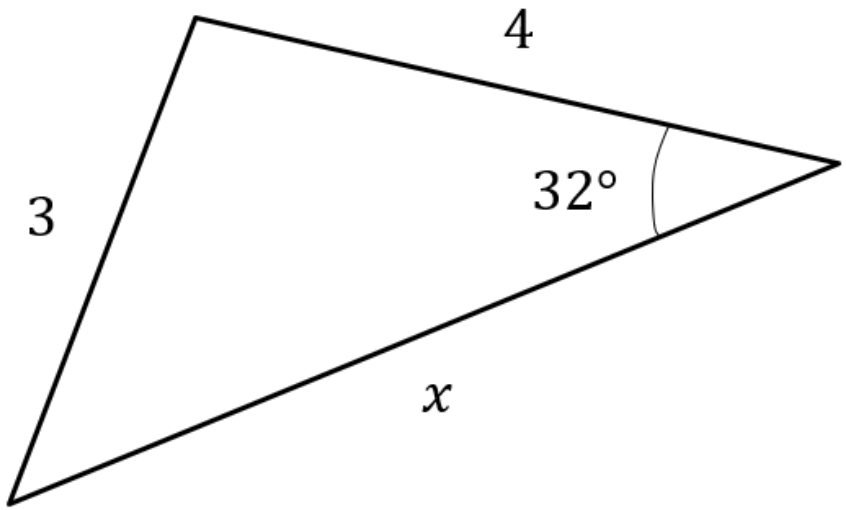
Worked example

Calculate the value of x



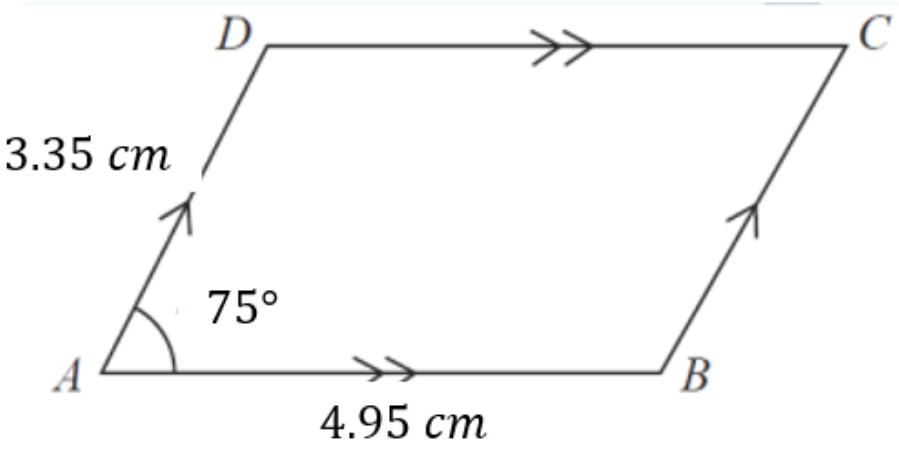
Your turn

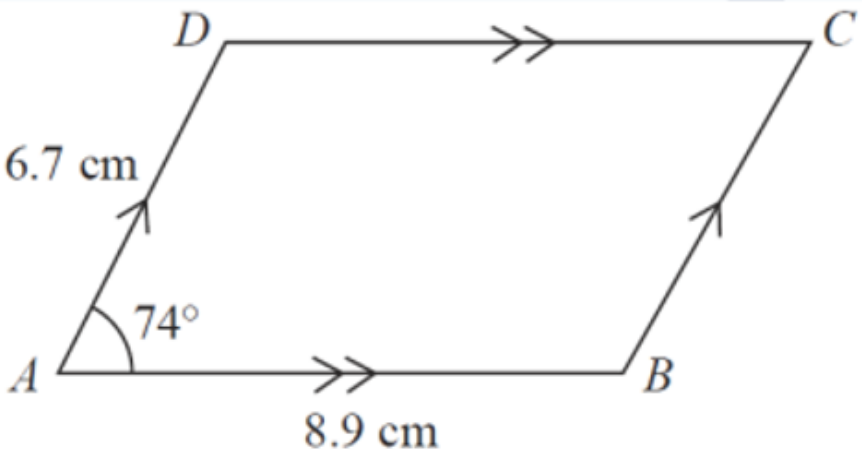
Calculate the value of x



$x = 5.52$ (3 sf)

Worked example	Your turn
Calculate the area of the parallelogram	Calculate the area of the parallelogram

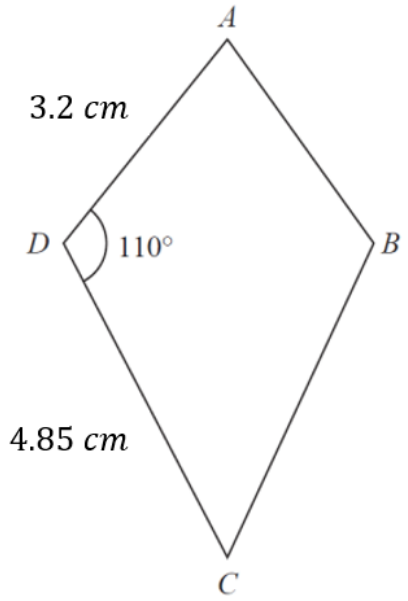




$57.32\ cm^2$ (2 dp)

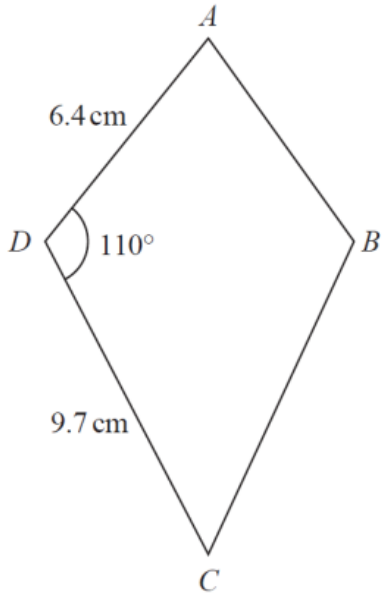
Worked example

Calculate the area of the kite



Your turn

Calculate the area of the kite

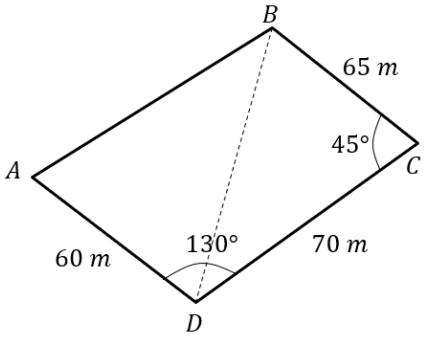


$58.34\ cm^2$ (2 dp)

Worked example

The diagram shows the locations of four mobile phone masts in a field.
$BC = 75\ m$, $CD = 80m$, angle $BCD = 55°$ and angle $ADC = 140°$.
In order that the masts do not interfere with each other, they must be at least 65m apart.
Given that A is the minimum distance from D, find:

a) The distance A is from B
b) The angle BAD
c) The area enclosed by the four masts.

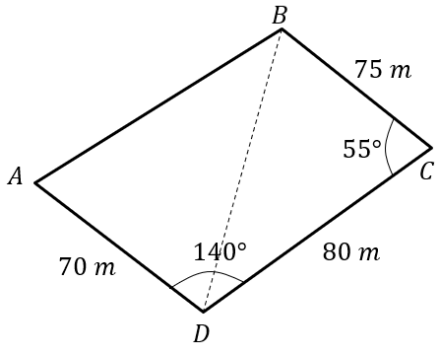


Your turn

The diagram shows the locations of four mobile phone masts in a field.
$BC = 75\ m$, $CD = 80m$, angle $BCD = 55°$ and angle $ADC = 140°$.
In order that the masts do not interfere with each other, they must be at least 70m apart.
Given that A is the minimum distance from D, find:

a) The distance A is from B
b) The angle BAD
c) The area enclosed by the four masts.

a) $9.21\ m$ (3 sf)
b) $50.3°$ (3 sf)
c) $4940\ m^2$ (3 sf)



9.5) Graphs of sine, cosine and tangent

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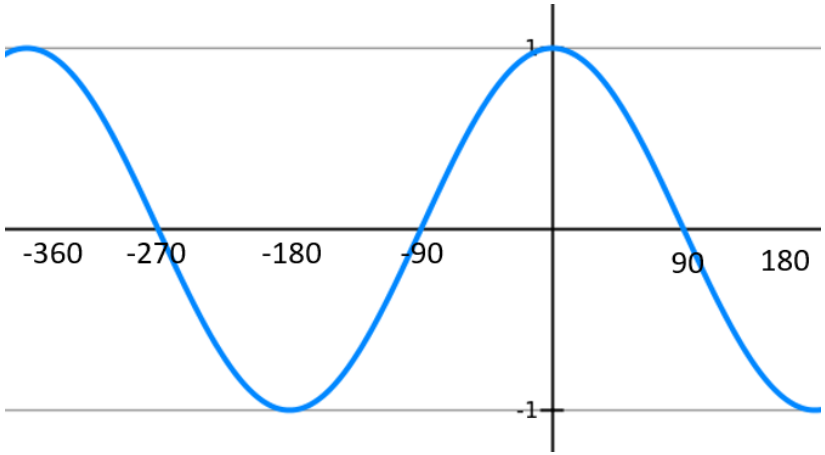
Worked example

Sketch the graph of $y = \sin x, -180 \leq x \leq 360°$

Your turn

Sketch the graph of $y = \cos x, -360 \leq x \leq 180°$

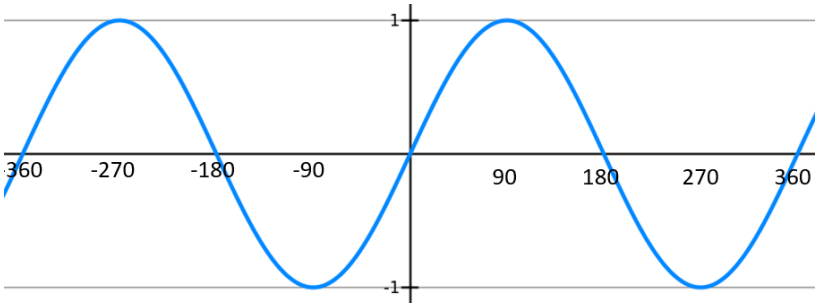


Worked example

A sketch of $y = \sin x\,, -360° \leq x \leq 360°$ is shown.

Given that $\sin 30 = \frac{1}{2}$, find:

a) $\sin(150°)$
b) $\sin(-300°)$
c) $\sin(330°)$
d) $\sin(-210°)$

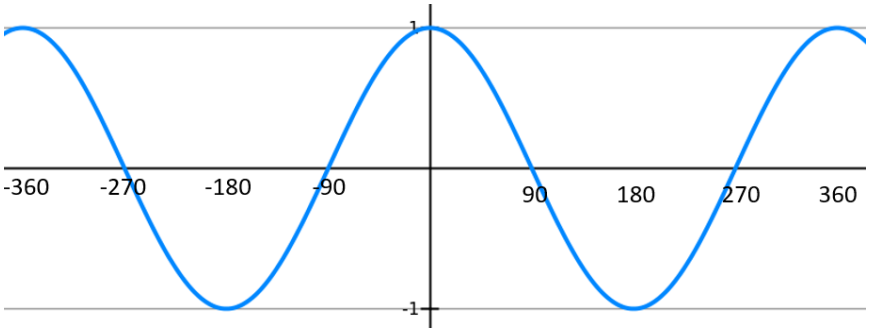


Your turn

A sketch of $y = \cos x\,, -360° \leq x \leq 360°$ is shown.

Given that $\cos 30 = \frac{\sqrt{3}}{2}$, find:

a) $\cos(-30°)$
b) $\cos(330°)$
c) $\cos(150°)$
d) $\cos(-210°)$



a) $\frac{\sqrt{3}}{2}$
b) $\frac{\sqrt{3}}{2}$
c) $-\frac{\sqrt{3}}{2}$
d) $-\frac{\sqrt{3}}{2}$

Worked example

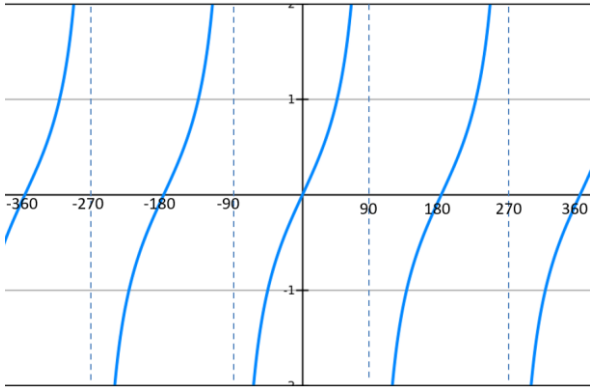
A sketch of $y = \tan x$, $-360° \leq x \leq 360°$ is shown.

Given that $\tan 60 = \sqrt{3}$, find:

a) $\tan(-60°)$

b) $\tan(-300°)$

c) $\tan(120°)$



Your turn

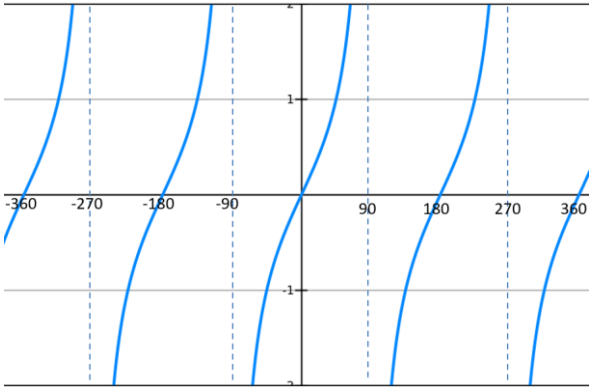
A sketch of $y = \tan x$, $-360° \leq x \leq 360°$ is shown.

Given that $\tan 30 = \frac{\sqrt{3}}{3}$, find:

a) $\tan(-30°)$

b) $\tan(-330°)$

c) $\tan(150°)$



a) $-\frac{\sqrt{3}}{3}$

b) $\frac{\sqrt{3}}{3}$

c) $-\frac{\sqrt{3}}{3}$

9.6) Transforming trigonometric graphs

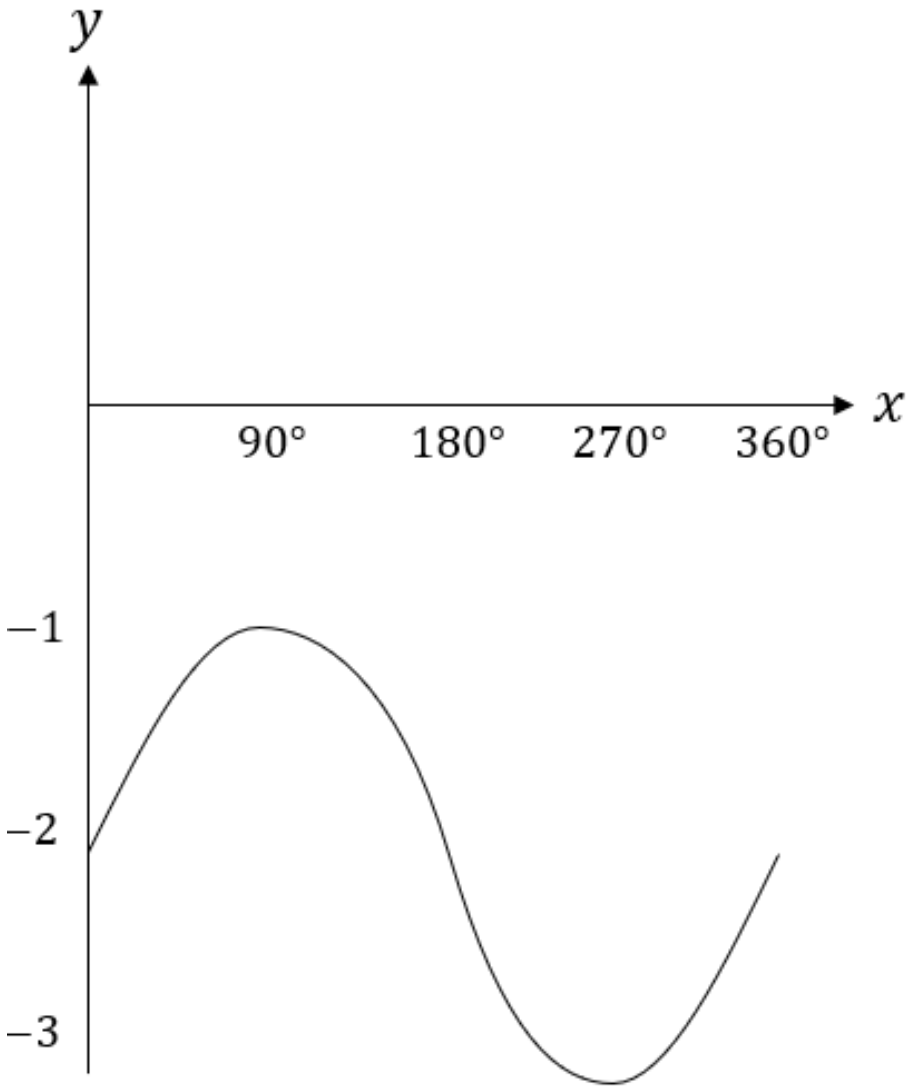
Chapter CONTENTS

Worked example

Sketch $y = \cos x + 1, 0 \leq x \leq 360°$

Your turn

Sketch $y = \sin x - 2, 0 \leq x \leq 360°$

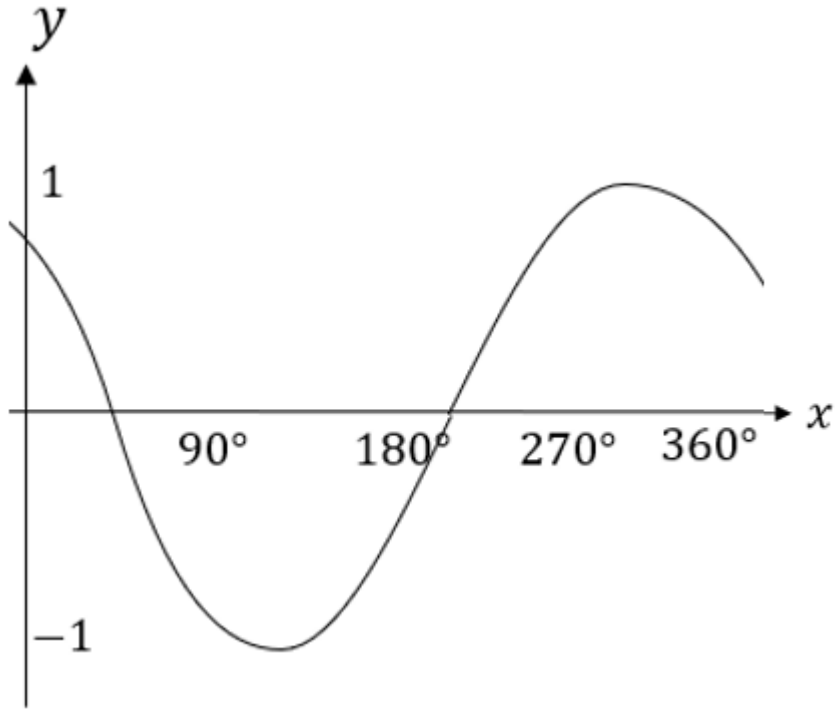


Worked example

Sketch $y = \sin(x - 45°), 0 \leq x \leq 360°$

Your turn

Sketch $y = \cos(x + 45°), 0 \leq x \leq 360°$

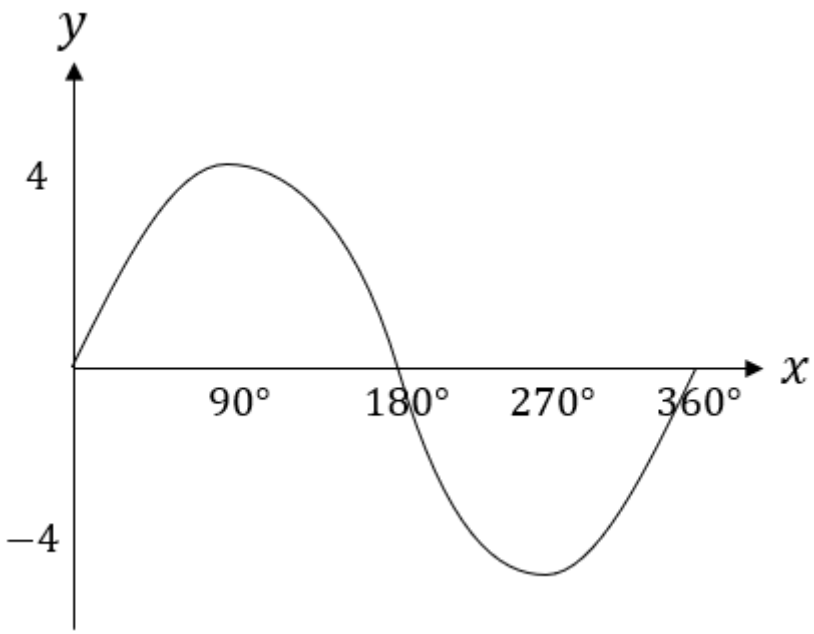


Worked example

Sketch $y = 3\cos x, 0 \leq x \leq 360°$

Your turn

Sketch $y = 4\sin x, 0 \leq x \leq 360°$



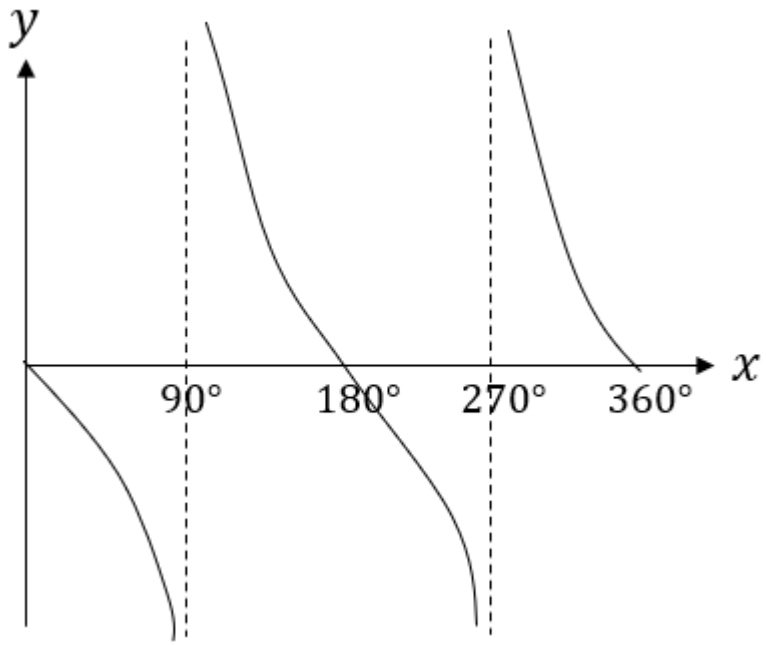
Worked example

Sketch $y = -\sin x, 0 \leq x \leq 360°$

Sketch $y = \cos(-x), 0 \leq x \leq 360°$

Your turn

Sketch $y = -\tan x, 0 \leq x \leq 360°$



Worked example

Sketch $y = \cos(2x), 0 \leq x \leq 360°$

Your turn

Sketch $y = \sin\left(\frac{x}{2}\right), 0 \leq x \leq 360°$