9) Constant acceleration

9.1) Displacement-time graphs

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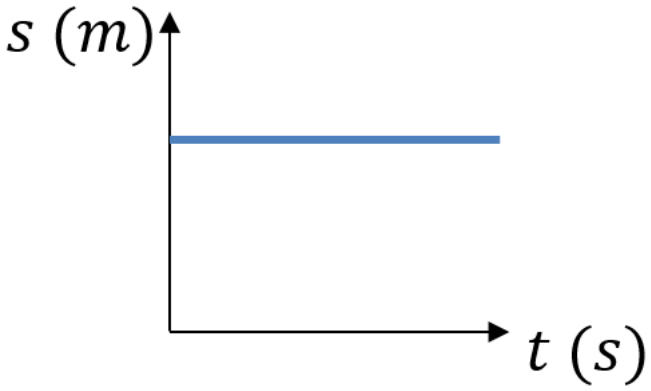
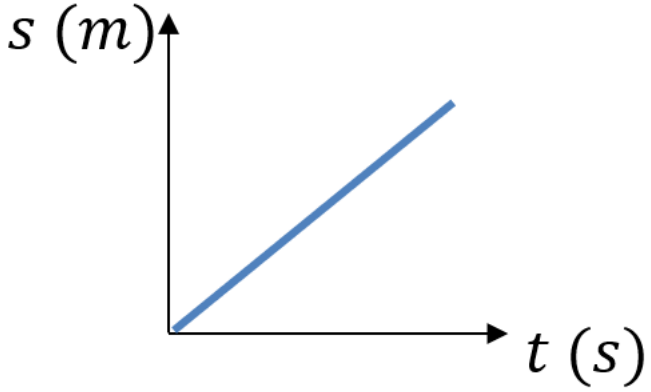
Worked example

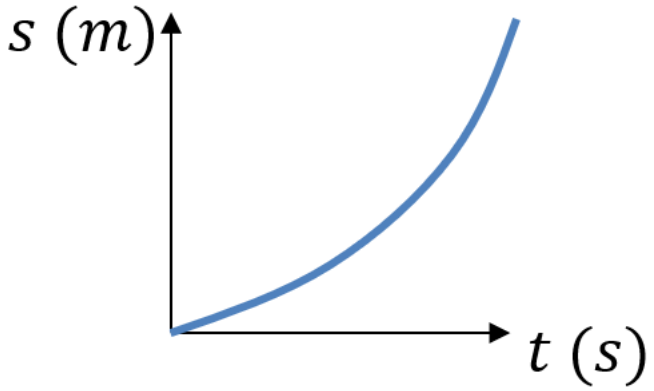
Describe the motion of each object from the displacement-time graph:





Your turn

Describe the motion of each object from the displacement-time graph:



Object is accelerating

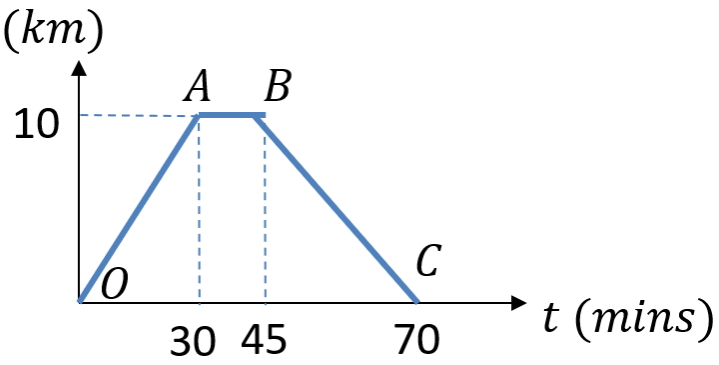
Worked example

A cyclist rides in a straight line for 30 minutes. She waits for a quarter of an hour, then returns in a straight line to her starting point in 25 minutes.

a) Work out the average velocity for each stage of the journey in km h^{-1}.

b) Write down the average velocity for the whole journey.

c) Work out average speed for the whole jo



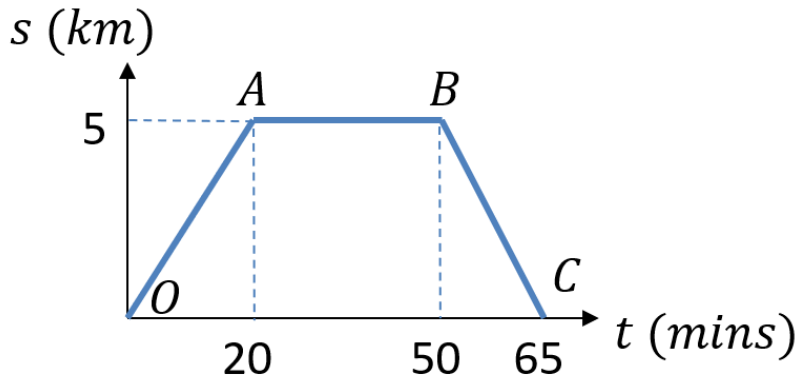
Your turn

A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes.

a) Work out the average velocity for each stage of the journey in km h^{-1}.

b) Write down the average velocity for the whole journey.

c) Work out average speed for the whole



a) OA: 15 $km\ h^{-1}$; AB: 0 $km\ h^{-1}$; BC: 20 $km\ h^{-1}$

b) 0

c) 9.23 $km\ h^{-1}$ $(3sf)$

9.2) Velocity-time graphs

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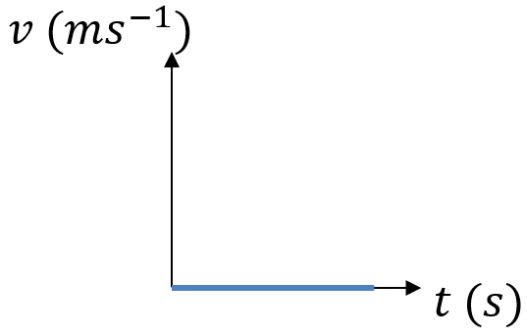
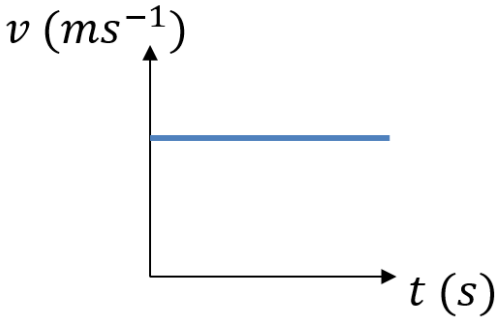
Worked example

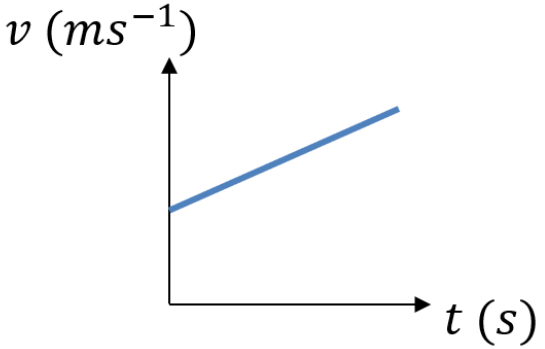
Describe the motion of each object from the velocity-time graph:





Your turn

Describe the motion of each object from the velocity-time graph:



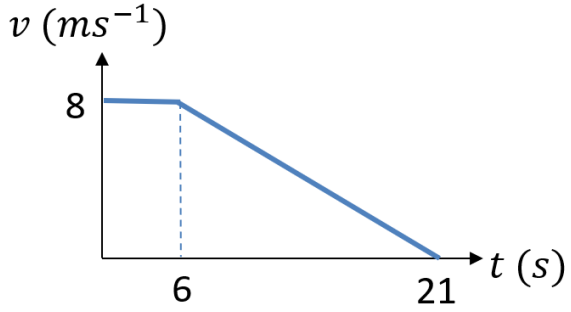
Object has constant acceleration. Velocity is increasing at a constant rate.

Worked example

A cyclist is moving along a straight road for a period of 21 seconds. For the first 6 seconds, she moves at a constant speed of 8 ms^{-1}. She then decelerates at a constant rate, stopping after a further 15 seconds.

(a) Find the displacement from the starting point of the cyclist after this 21 second period.

(b) Work out the rate at which the cyclist decelerates.

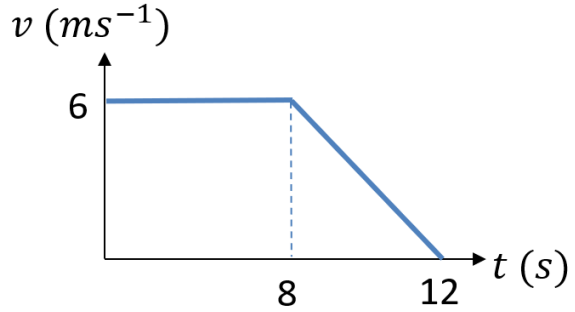


Your turn

A cyclist is moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds.

(a) Find the displacement from the starting point of the cyclist after this 12 second period.

(b) Work out the rate at which the cyclist decelerates.



a) 60 m

b) 1.5 ms^{-2}

Diagrams/Graphs used with permission from DrFrostMaths: https://www.drfrostmaths.com/

Worked example

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 16 ms^{-1} in T seconds. The particle then travels at a constant velocity of 16 ms^{-1} for $3T$ seconds. The particle then decelerates uniformly to rest in a further 4 s.

(a) Sketch a velocity-time graph to illustrate the motion of the particle.

Give then the total displacement of the particle is 592m.

(b) find the value of T.

Your turn

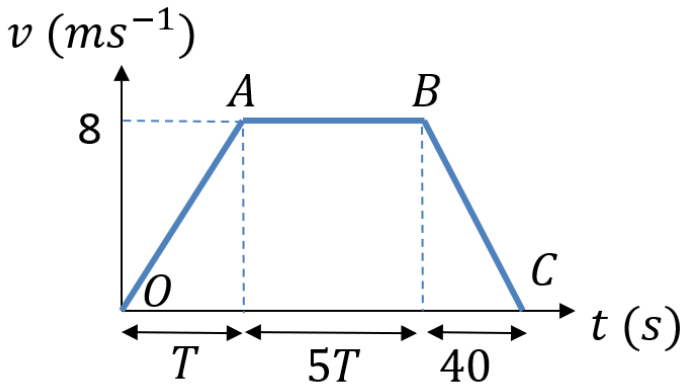
A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 ms^{-1} in T seconds. The particle then travels at a constant velocity of 8 ms^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

(a) Sketch a velocity-time graph to illustrate the motion of the particle.

Give then the total displacement of the particle is 600m.

(b) find the value of T.

a)



b) $T = 10$

Worked example

A car is travelling along a straight horizontal road. The car takes $60\ s$ to travel between two sets of traffic lights which are $1072.5\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $15\ s$ until its speed is $11\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights

b) Find the value of T

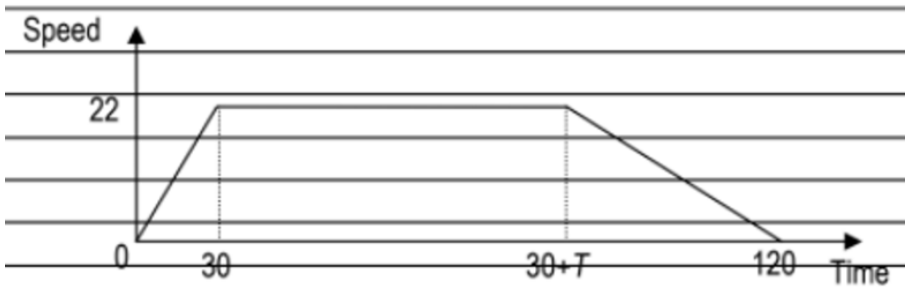
Your turn

A car is travelling along a straight horizontal road. The car takes $120\ s$ to travel between two sets of traffic lights which are $2145\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $30\ s$ until its speed is $22\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights

b) Find the value of T

a)



b) $T = 75$

Diagrams/Graphs used with permission from DrFrostMaths: https://www.drfrostmaths.com/

Worked example

A car is travelling along a straight horizontal road. The car takes $60\ s$ to travel between two sets of traffic lights which are $1072.5\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $15\ s$ until its speed is $11\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights

b) Find the value of T

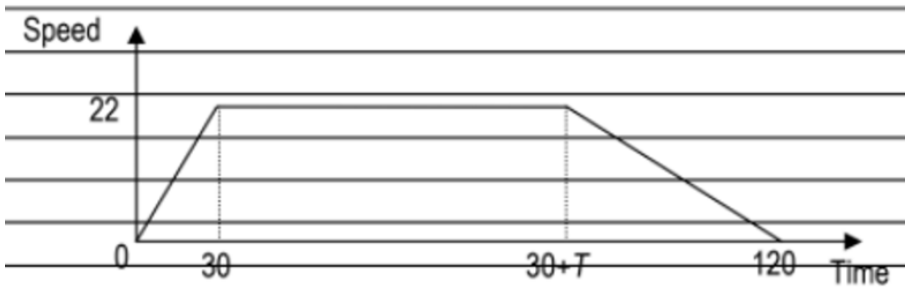
Your turn

A car is travelling along a straight horizontal road. The car takes $120\ s$ to travel between two sets of traffic lights which are $2145\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $30\ s$ until its speed is $22\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights

b) Find the value of T

a)



b) $T = 75$

Diagrams/Graphs used with permission from DrFrostMaths: https://www.drfrostmaths.com/

Worked example

A car is travelling along a straight horizontal road. The car takes $60\ s$ to travel between two sets of traffic lights which are $1072.5\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $15\ s$ until its speed is $11\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.
A motorcycle leaves the first set of traffic lights 15 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, and passes the car at the point A which is $495\ m$ from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed $11\ ms^{-1}$

c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A

Your turn

A car is travelling along a straight horizontal road. The car takes $120\ s$ to travel between two sets of traffic lights which are $2145\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $30\ s$ until its speed is $22\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, and passes the car at the point A which is $990\ m$ from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed $22\ ms^{-1}$

c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A

c) 50 seconds

Worked example

A car is moving along a straight horizontal road.
At time $t = 0$, the car is moving with speed $10\ ms^{-1}$ and is at the point A. The car maintains this speed for $50\ s$.
The car then moves with constant deceleration $0.6\ ms^{-2}$, reducing its speed from $10\ ms^{-1}$ to $4\ ms^{-1}$.
The car then moves with constant speed $4\ ms^{-1}$ for $30\ s$.
The car then moves with constant acceleration until it is moving with speed $10\ ms^{-1}$ at the point B.
Given that the distance from A to B is $980\ m$, find the time taken for the car to move from A to B

Your turn

A car is moving along a straight horizontal road.
At time $t = 0$, the car is moving with speed $20\ ms^{-1}$ and is at the point A. The car maintains this speed for $25\ s$.
The car then moves with constant deceleration $0.4\ ms^{-2}$, reducing its speed from $20\ ms^{-1}$ to $8\ ms^{-1}$.
The car then moves with constant speed $8\ ms^{-1}$ for $60\ s$.
The car then moves with constant acceleration until it is moving with speed $20\ ms^{-1}$ at the point B.
Given that the distance from A to B is $1960\ m$, find the time taken for the car to move from A to B

155 seconds

9.3) Constant acceleration formulae 1

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Worked example

A cyclist is travelling along a straight road.
She accelerates at a constant rate from a velocity of 5 ms^{-1} to a velocity of 7.4 ms^{-1} in 50 seconds.
Find:
(a) the distance she travels in these 50 seconds
(b) her acceleration in these 50 seconds.

Your turn

A cyclist is travelling along a straight road.
She accelerates at a constant rate from a velocity of 4 ms^{-1} to a velocity of 7.5 ms^{-1} in 40 seconds.
Find:
(a) the distance she travels in these 40 seconds
(b) her acceleration in these 40 seconds.

a) $230\ m$
b) $0.0875\ ms^{-2}$

Worked example

A particle moves in a straight line from a point A to a point B with a constant deceleration $3\ ms^{-2}$. The velocity of the particle at A is $16\ ms^{-1}$ and the velocity of the particle at B is $4\ ms^{-1}$. Find:

(a) the time taken for the particle to move from A to B.

(b) the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration $3\ ms^{-2}$.
The particle is at the point C 12 seconds after passing through the point A. Find:

(c) the velocity of the particle at C.

(d) The distance from A to C.

Your turn

A particle moves in a straight line from a point A to a point B with a constant deceleration $1.5\ ms^{-2}$. The velocity of the particle at A is $8\ ms^{-1}$ and the velocity of the particle at B is $2\ ms^{-1}$. Find:

(a) the time taken for the particle to move from A to B.

(b) the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration $1.5\ ms^{-2}$.
The particle is at the point C 6 seconds after passing through the point A. Find:

(c) the velocity of the particle at C.

(d) The distance from A to C.

a) 4 s

b) $20\ m$

c) $1\ ms^{-1}$ in the direction $\overrightarrow{BA}$

d) $21\ m$

Worked example

A car moves from traffic lights along a straight road with constant acceleration.
The car starts from rest at the traffic lights and 20 seconds later the car passes a speed-trap where it is registered as travelling at $54\ km\ h^{-1}$. Find:

(a) the acceleration of the car
(b) the distance between the traffic lights and the speed-trap.

Your turn

A car moves from traffic lights along a straight road with constant acceleration.
The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at $45\ km\ h^{-1}$. Find:

(a) the acceleration of the car
(b) the distance between the traffic lights and the speed-trap.

a) $\frac{5}{12}\ ms^{-2} = 0.417\ ms^{-2}$ (3 sf)

b) $187.5\ m$

9.4) Constant acceleration formulae 2

Chapter CONTENTS

Worked example

Use the equations $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$ to derive:

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Your turn

Use the equations $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$ to derive:

$$s = ut + \frac{1}{2}at^2$$

Shown

Worked example

A particle is moving along a straight line from A to B with constant acceleration $3\ ms^{-2}$.
The velocity of the particle is $5\ ms^{-1}$ in the direction $\overrightarrow{AB}$.
The velocity of the particle at B is $81\ ms^{-1}$ in the same direction.
Find the distance from A to B.

Your turn

A particle is moving along a straight line from A to B with constant acceleration $5\ ms^{-2}$.
The velocity of the particle is $3\ ms^{-1}$ in the direction $\overrightarrow{AB}$.
The velocity of the particle at B is $18\ ms^{-1}$ in the same direction.
Find the distance from A to B.

$31.5\ m$

Worked example

A particle is moving in a straight horizontal line with constant deceleration 6 ms^{-2}.
At time $t = 0$ the particle passes through a point O with speed 23 ms^{-1} travelling towards a point A, where $OA = 40$ m. Find:
(a) the times when the particle passes through A
(b) the value of t when the particle returns to O.

Your turn

A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2}.
At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1} travelling towards a point A, where $OA = 20$ m. Find:
(a) the times when the particle passes through A
(b) the value of t when the particle returns to O.

a) $t = 2.5$ s, $t = 4$ s
b) $t = 6.5$ s

Worked example

A particle is moving in a straight horizontal line with constant deceleration 6 ms^{-2}.
At time $t = 0$ the particle passes through a point O with speed 23 ms^{-1}.
Find the total distance travelled by the particle between when it first passes O and returns to O

Your turn

A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2}.
At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1}.
Find the total distance travelled by the particle between when it first passes O and returns to O

$42.25\ m$

Worked example

Two particles P and Q are moving along the same straight horizontal line with constant accelerations 2 and 4 ms^{-2} respectively. At time $t = 0$, P passes through a point A with speed 12 ms^{-1}. One second later Q passes through A with speed 6 ms^{-1}, moving in the same direction as P.

a) Find the value of t where the particles meet.

b) Find the distance of A from the point where the particles meet.

Your turn

Two particles P and Q are moving along the same straight horizontal line with constant accelerations 6 and 8 ms^{-2} respectively. At time $t = 0$, P passes through a point A with speed 10 ms^{-1}. One second later Q passes through A with speed 5 ms^{-1}, moving in the same direction as P.

a) Find the value of t where the particles meet.

b) Find the distance of A from the point where the particles meet.

a) $t = 13.1$ s (3 sf)

b) 644 m (3 sf)

Worked example

A particle moves in a straight horizontal line with constant acceleration from A to B, then B to C.
AB = 3 km and BC = 12 km.
It takes 2 hour from A to B and 4 hours from B to C.
Find:

a) The acceleration of the particle
b) The particle's speed as it passes A

Your turn

A particle moves in a straight horizontal line with constant acceleration from A to B, then B to C.
AB = 4 km and BC = 12 km.
It takes 2 hours from A to B and 3 hours from B to C.
Find:

a) The acceleration of the particle
b) The particle's speed as it passes A

a) $0.8\ km\ h^{-2} = 6.1728 \times 10^{-5}\ ms^{-2}$ (3 sf)
b) $1.2\ km\ h^{-1} = 0.333\ ms^{-1}$ (3 sf)

9.5) Vertical motion under gravity

Chapter CONTENTS

Worked example

A book falls off the top shelf of a bookcase. The shelf is 2.8 m above a wooden floor. Find:

(a) the time the book takes to reach the floor,

(b) the speed with which the book strikes the floor.

Your turn

A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find:

(a) the time the book takes to reach the floor,

(b) the speed with which the book strikes the floor.

a) $0.53\ s$

b) $5.2\ ms^{-1}$

Worked example

A ball is projected vertically upwards, from a point X which is 5m above the ground, with speed 15 ms^{-1}. Find

(a) the greatest height above the ground reached by the ball,

(b) the time of flight of the ball

Your turn

A ball is projected vertically upwards, from a point X which is 7m above the ground, with speed 21 ms^{-1}. Find

(a) the greatest height above the ground reached by the ball,

(b) the time of flight of the ball

a) $30\ m$ (2 sf)

b) $4.6\ s$ (2 sf)

Worked example

A ball is projected vertically upwards from ground level at a speed of 40 ms^{-1}.
Determine the amount of time the ball is at least 20m above ground level.

Your turn

A ball is projected vertically upwards from ground level at a speed of 20 ms^{-1}.
Determine the amount of time the ball is at least 10m above ground level.

$2.9\ s$ (2 sf)

Worked example

A ball is projected vertically upwards with initial speed of 20 ms^{-1}.
It hits the ground 5 s later.
Find the height above the ground from which the ball was thrown.

Your turn

A ball is projected vertically upwards with initial speed of 15 ms^{-1}.
It hits the ground 5 s later.
Find the height above the ground from which the ball was thrown.

$47.5\ m$

Worked example

A stone is thrown vertically upward from a point which is $8\ m$ above the ground with speed $5\ ms^{-1}$.
Find:

a) The time of flight of the stone
b) The total distance travelled by the stone

Your turn

A stone is thrown vertically upward from a point which is $5\ m$ above the ground with speed $8\ ms^{-1}$.
Find:

a) The time of flight of the stone
b) The total distance travelled by the stone

a) 2.1 s (2 sf)
b) $12\ m$ (2 sf)

Worked example

Ball A falls vertically from rest from the top of a tower $48\ m$ high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed $24\ ms^{-1}$. The balls collide. Find the distance to the point where the balls collide from the bottom of the tower.

Your turn

Ball A falls vertically from rest from the top of a tower $63\ m$ high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed $21\ ms^{-1}$. The balls collide. Find the distance to the point where the balls collide from the bottom of the tower.

$19\ m$ (2 sf)

Worked example

At time $t = 0$, two balls A and B are projected vertically upwards. Ball A is projected upwards with speed $3\ ms^{-1}$ from a point $40\ m$ above the horizontal ground. Ball B is projected vertically upwards from the ground with speed $30\ ms^{-1}$. The balls are modelled as particles moving freely under gravity. Find the time and the height at which the balls are at the same vertical height.

Your turn

At time $t = 0$, two balls A and B are projected vertically upwards. Ball A is projected upwards with speed $2\ ms^{-1}$ from a point $50\ m$ above the horizontal ground. Ball B is projected vertically upwards from the ground with speed $20\ ms^{-1}$. The balls are modelled as particles moving freely under gravity. Find the time and the height at which the balls are at the same vertical height.

$t = 2.8\ s\ (2\ \text{sf})$

$h = 18\ m\ (2\ \text{sf})$

Worked example

A ball is released from rest at a point which is $20\ m$ above a wooden floor. Each time the ball strikes the floor, it rebounds with $\frac{2}{3}$ of the speed with which it strikes the floor.
Find the greatest height above the floor reached by the ball:

a) The first time it rebounds from the floor
b) The second time it rebounds from the floor.

Your turn

A ball is released from rest at a point which is $10\ m$ above a wooden floor. Each time the ball strikes the floor, it rebounds with $\frac{3}{4}$ of the speed with which it strikes the floor.
Find the greatest height above the floor reached by the ball:

a) The first time it rebounds from the floor
b) The second time it rebounds from the floor.

a) $5.6\ m$ (2 sf)
b) $3.2\ m$ (2 sf)