8.5) Integrating vectors

Worked example

A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by

$$\boldsymbol{v} = 2t\boldsymbol{i} + \frac{1}{3}t^2\boldsymbol{j}, \qquad t \geq 0$$

When $t = 0$, the position vector of P with respect to a fixed O is $(5\boldsymbol{i} - 4\boldsymbol{j})$ m.

Find the position vector of P at time t seconds.

Your turn

A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by

$$\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}, \qquad t \geq 0$$

When $t = 0$, the position vector of P with respect to a fixed O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m.

Find the position vector of P at time t seconds.

$$\left(\left(\frac{3t^2}{2} + 2\right)\boldsymbol{i} + \left(\frac{t^3}{6} - 3\right)\boldsymbol{j}\right)m$$

Worked example

A particle P is moving in a plane so that, at time t seconds, its acceleration is $(3\boldsymbol{i} - 4t\boldsymbol{j})\ ms^{-2}$.
When $t = 2$, the velocity of P is $-3\boldsymbol{j}\ ms^{-1}$ and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find:

(a) the angle between the direction of motion of P and $\boldsymbol{j}$ when $t = 3$

(b) the distance of P from O when $t = 0$.

Your turn

A particle P is moving in a plane so that, at time t seconds, its acceleration is $(4\boldsymbol{i} - 2t\boldsymbol{j})\ ms^{-2}$.
When $t = 3$, the velocity of P is $6\boldsymbol{i}\ ms^{-1}$ and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find:

(a) the angle between the direction of motion of P and $\boldsymbol{i}$ when $t = 2$

(b) the distance of P from O when $t = 0$.

a) $68.2°$ (1 dp)

b) $25\ m$

Worked example

The velocity of a particle P at time t seconds is $\left((6t^2 - 4)\boldsymbol{i} + 10\boldsymbol{j}\right) ms^{-1}$.
When $t = 0$, the position vector of P with respect to a fixed origin O is $(5\boldsymbol{i} - 3\boldsymbol{j})\ m$.
A second particle Q moves with constant velocity $(3\boldsymbol{i} + 5\boldsymbol{j})\ ms^{-1}$.
When $t = 0$, the position vector of Q with respect to the fixed origin O is $2\boldsymbol{j}\ m$.
Prove that P and Q collide.

Your turn

The velocity of a particle P at time t seconds is $\left((3t^2 - 8)\boldsymbol{i} + 5\boldsymbol{j}\right) ms^{-1}$.
When $t = 0$, the position vector of P with respect to a fixed origin O is $(2\boldsymbol{i} - 4\boldsymbol{j})\ m$.
A second particle Q moves with constant velocity $(8\boldsymbol{i} + 4\boldsymbol{j})\ ms^{-1}$.
When $t = 0$, the position vector of Q with respect to the fixed origin O is $2\boldsymbol{i}\ m$.
Prove that P and Q collide.

Proof