8.4) Points of intersection

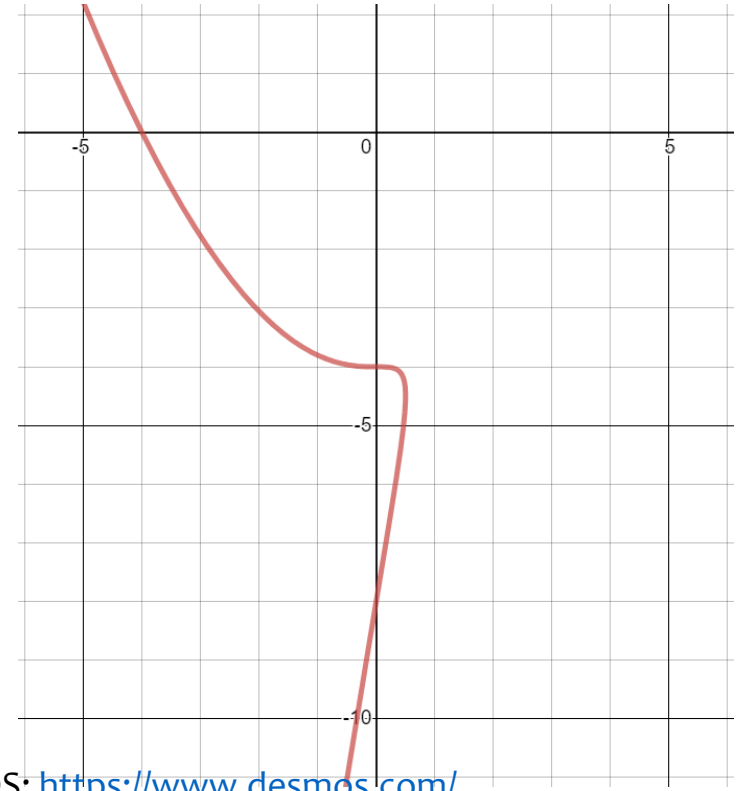
Worked example

A curve C is given by the parametric equations
$x = at^2 + t, \quad y = a(t^3 + 27), \quad t \in \mathbb{R},$
where a is a non-zero constant.
Given that C passes through the point $(-6,0)$,
a) find the value of a.
b) find the coordinates of the points A and B where the curve crosses the y-axis.

Your turn

A curve C is given by the parametric equations
$x = at^2 + t, \quad y = a(t^3 + 8), \quad t \in \mathbb{R},$
where a is a non-zero constant.
Given that C passes through the point $(-4,0)$,
a) find the value of a.
b) find the coordinates of the points A and B where the curve crosses the y-axis.

a) $a = -\frac{1}{2}$
b) $(0, -4)$ and $(0, -8)$



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Worked example

A curve C is given by the parametric equations

$$x = t^2,\ y = 2t,\ \ t \in \mathbb{R}$$

Find the coordinates of the point(s) of intersection between the curve C and the line $x + y - 8 = 0$

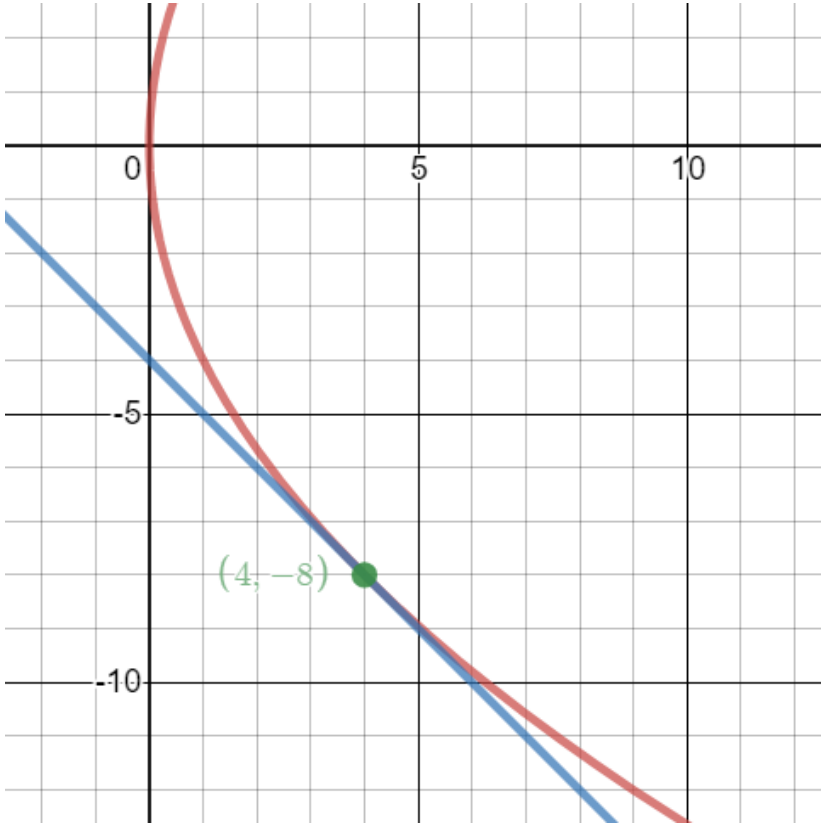
Your turn

A curve C is given by the parametric equations

$$x = t^2,\ y = 4t,\ \ t \in \mathbb{R}$$

Find the coordinates of the point(s) of intersection between the curve C and the line $x + y + 4 = 0$

$(4, -8)$



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Worked example

A curve C is given by the parametric equations

$$x = \cos t - \sin t, \qquad y = \left(t + \frac{\pi}{6}\right)^2, \qquad -\frac{\pi}{3} < t < \frac{3\pi}{2}$$

a) Find the point where the curve intersects the line $y = \pi^2$.

b) Find the coordinates of the points where the curve cuts the y-axis.

Your turn

A curve C is given by the parametric equations

$$x = \cos t + \sin t, \qquad y = \left(t - \frac{\pi}{6}\right)^2, \qquad -\frac{\pi}{2} < t < \frac{4\pi}{3}$$

a) Find the point where the curve intersects the line $y = \pi^2$.

b) Find the coordinates of the points where the curve cuts the y-axis.

a) $\left(-\frac{1+\sqrt{3}}{2}, \pi^2\right)$

b) $\left(0, \frac{25\pi^2}{144}\right)$ and $\left(0, \frac{49\pi^2}{144}\right)$

Worked example

A curve C is given by the parametric equations

$$x = 1 - \frac{1}{3}t, \qquad y = 3^t - 1, \qquad t \in \mathbb{R}$$

Find the coordinates of the x and y intercepts

Your turn

A curve C is given by the parametric equations

$$x = 1 - \frac{1}{2}t, \qquad y = 2^t - 1, \qquad t \in \mathbb{R}$$

Find the coordinates of the x and y intercepts

$(0, 3)$ and $(1, 0)$

Worked example

A curve C is given by the parametric equations

$$x = e^{3t}, \qquad y = e^{t} + 1, \qquad t \in \mathbb{R}$$

A straight line l passes through the points A and B where $t = \ln 3$ and $t = \ln 4$ respectively.

Find an equation for l in the form $ax + by + c = 0$

Your turn

A curve C is given by the parametric equations

$$x = e^{2t}, \qquad y = e^{t} - 1, \qquad t \in \mathbb{R}$$

A straight line l passes through the points A and B where $t = \ln 2$ and $t = \ln 3$ respectively.

Find an equation for l in the form $ax + by + c = 0$

$$x - 5y + 1 = 0$$