8D Coupled Differential Equations

$$\frac{dx}{dt} = ax + by + f(t)$$

$$\frac{dy}{dt} = cx + dy + g(t)$$

1. At the start of 2010, a survey began on the number of bears and fish on a remote island in Northern Canada. After t years the number of bears, x, and the number of fish, y, in the area are modelled by the differential equations:

$$\frac{dx}{dt} = 0.3x + 0.1y$$

$$\frac{dy}{dt} = -0.1x + 0.5y$$

a) Show that $\frac{d^2x}{dt^2} - 0.8\frac{dx}{dt} + 0.16x = 0$

b) Find the general solution for the number of bears on the island at time t

c) Find the general solution for the number of fish on the island at time t

d) At the start of 2010, there were 5 bears and 20 fish on the island. Use this information to predict the number of bears on the island in 2020.

e) Comment on the suitability of the model

no other factors were taken into account as well – the f(t) and g(t) mentioned at the start

2. Two barrels contain contaminated water. At time t seconds, the amount of contaminant in barrel A is x ml and the amount in barrel B is y ml. Additional contaminated water flows into barrel A at a rate of 5 ml per second. Contaminated water flows from barrel A to barrel B and then back to barrel A through two connecting hoses, and then drains out of barrel A to leave the system completely. The system is modelled using the following differential equations:

$$\frac{dx}{dt} = 5 + \frac{4}{9}y - \frac{1}{7}x$$

$$\frac{dy}{dt} = \frac{3}{70}x - \frac{4}{9}y$$

Show that $630\frac{d^2y}{dt^2} + 370\frac{dy}{dt} + 28y = 135$