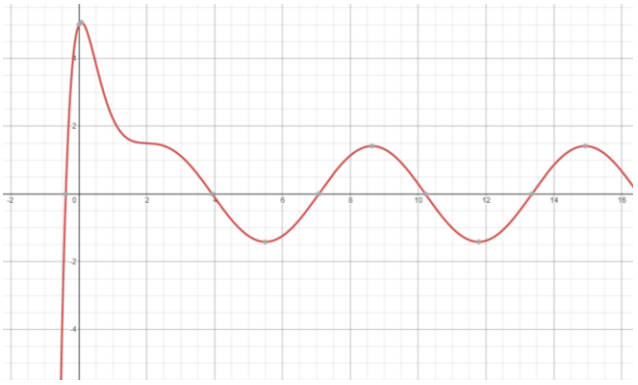
# 8C Part 2 Forced Harmonic Motion

1. A particle $P$ of mass 1.5kg is moving along the x-axis. At time $t$ the displacement of $P$ from the origin $O$ is $x$ metres and the speed of $P$ is $v\,ms^{-1}$. Three forces act on $P$, namely a restoring force of $7.5x\,N$, a resistance to motion of $P$ of magnitude $6v\,N$ and a force of magnitude $12sint\,N$ acting in the direction $OP$. When $t = 0$, $x = 5$ and $\frac{dx}{dt} = 2$.

a) Show that $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 8sint$

b) Find $x$ as a function of $t$

c) Describe the motion when $t$ is large



$$\boldsymbol{x = e^{-2t}(6cost + 13sint) + sint - cost}$$

2. A particle $P$ is attached to end $A$ of a light elastic string $AB$. Initially the particle and the string lie at rest on a smooth horizontal plane. At time $t = 0$, the end $B$ of the string is set into motion and moves with constant speed $U$ in the direction $AB$, and the extension in the string is $x$. Air resistance acting on $P$ is proportional to its speed. The subsequent motion can be modelled by the differential equation:

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + k^2x = 2kU$$

Find an expression for $x$ in terms of $U$, $k$ and $t$.