**8C Part 2 Forced Harmonic Motion**

1. A particle $P$ of mass 1.5kg is moving along the x-axis. At time $t$ the displacement of $P$ from the origin $O$ is $x$ metres and the speed of $P$ is $v ms^{-1}$. Three forces act on $P$, namely a restoring force of $7.5x N$, a resistance to motion of $P$ of magnitude $6v N$ and a force of magnitude $12sint N$ acting in the direction $OP$. When $t=0$, $x=5$ and $\frac{dx}{dt}=2$.
2. Show that $\frac{d^{2}x}{dt^{2}}+4\frac{dx}{dt}+5x=8sint$
3. Find $x$ as a function of $t$
4. Describe the motion when $t$ is large



$$x=e^{-2t}\left(6cost+13sint\right)+sint-cost$$

1. A particle $P$ is attached to end $A$ of a light elastic string $AB$. Initially the particle and the string lie at rest on a smooth horizontal plane. At time $t=0$, the end $B$ of the string is set into motion and moves with constant speed $U$ in the direction $AB$, and the extension in the string is $x$. Air resistance acting on $P$ is proportional to its speed. The subsequent motion can be modelled by the differential equation:

$$\frac{d^{2}x}{dt^{2}}+2k\frac{dx}{dt}+k^{2}x=2kU$$

Find an expression for $x$ in terms of $U$, $k$ and $t$.