Forced Harmonic Motion

In addition to the ‘natural’ forces acting on the particle, i.e. damping force and restoring force, there may be a further a further force acting on the particle. This is known as **forced harmonic motion**.

All structures have natural frequencies of vibration. If an external agent causes them to vibrate at or close to one of these frequencies it can create resonance which can have devastating effects. Engineers must be able to predict these natural frequencies.

Forced harmonic motion

We can solve problems like this using the Non-homogeneous DE method.

Example

A particle of mass 1.5 kg is moving on the -axis. At time the displacement of from the origin is metres and the speed of is ms-1. Three forces act on , namely a restoring force of magnitude N, a resistance to the motion of of magnitude N and a force of magnitude N acting in the direction . When and .

1. Show that
2. Find as a function of .

Describe the motion when is large.

2. A particle is attached to end of a light elastic spring . Initially the particle and the string lie at rest on a smooth horizontal plane. At time , the end of the string is set in motion and moves with constant speed in the direction , and the displacement of from is . Air resistance acting on is proportional to its speed. The subsequent motion can be modelled by the differential equation

Find an expression for in terms of and

Ex 8c