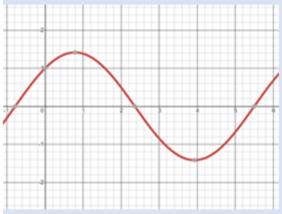
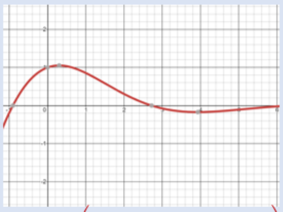
8C Part 1 Damped Harmonic Motion

$$\frac{d^2x}{dt^2} + x = 0$$



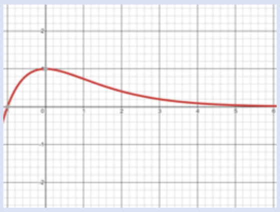
$x = cost + sint$

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$$



$x = e^{-\frac{1}{2}t}\left(cos\left(\frac{\sqrt{3}}{2}t\right) + sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$



$x = (1 + t)e^{-t}$

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = 0$$

$x = e^{\left(\frac{-3+\sqrt{5}}{2}\right)t} + e^{\left(\frac{-3-\sqrt{5}}{2}\right)t}$

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \omega^2 x = 0$$

If $k^2 < 4\omega^2$, then there will

If $k^2 = 4\omega^2$, then there will

If $k^2 > 4\omega$, then there will

1. A particle P of mass 0.5kg moves in a horizontal straight line. At time t seconds, the displacement of P from a fixed point O, on the line is $x\ m$ and the velocity of P is $v\ ms^{-1}$. A force of magnitude $8x\ N$ acts on P in the direction PO. The particle is also subject to a resistance of magnitude $4v\ N$. When $t = 0$, $x = 1.5$ and P is moving in the direction of x increasing with speed $4\ ms^{-1}$.

a) Show that $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 0$

b) Find the value of x when $t = 1$

2. A particle P hangs freely in equilibrium attached to one end of a light elastic string. The other end of the string is attached to a fixed point A. The particle is pulled down and held at rest in a container of liquid which exerts a resistance on the motion on P. P is then released from rest. While the string remains taut and the particle in the liquid, the motion can be modelled using the equation:

$$\frac{d^2x}{dt^2} + 6k\frac{dx}{dt} + 5k^2x = 0$$

Where k is a positive real constant.

Find the general solution to the differential equation and state the type of damping the particle is subject to.

3. One end of a light elastic spring is attached to a fixed point A. A particle P is attached to the other end and hangs in equilibrium vertically below A. The particle is pulled vertically down from its equilibrium position and released from rest. A resistance proportional to the speed of P acts on P.

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 2k^2x = 0$$

The equation of motion of P is given as:

Where k is a positive real constant and x is the displacement of P from its equilibrium position.

a) Find the general solution to the differential equation.

b) Find the period of the motion