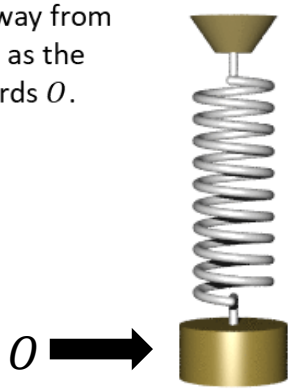
Simple Harmonic Motion

Simple Harmonic Motion (SHM) is motion in which the acceleration of a particle P is always towards a fixed point O on the line of motion of P. The **acceleration is proportional to the displacement x** of P from O.

We can see that when the particle is moving away from O, it is decelerating, as the acceleration is towards O.



Because of the compression/extension of the spring, as we double the displacement from O, we double the acceleration towards O, i.e. the acceleration is not constant (as it would be if acting under gravity).

Simple Harmonic Motion:

General solution $x = Asin\omega t + Bcos\omega t$

Writing in harmonic form: $x = \mathrm{asin}(\omega t + \alpha)$

So, the general solution of SHM can be expressed as a sine function from which we can deduce:

1) The solution varies between a and –a **Amplitude**
2) The solution is periodic with **Period** $\frac{2\pi}{\omega}$
3) The velocity and acceleration can be found by differentiating the solution with respect to t.

Example

A particle is moving along a straight line. At time t seconds its displacement, x m from a fixed point O is such that $\frac{d^2x}{dt^2} = -4x$.

Given that at $t = 0$, $x = 1$ and the particle is moving with velocity 4 ms^{-1},

(a) find an expression for the displacement of the particle after t seconds

(b) hence determine the maximum displacement of the particle from O.

Example

A particle P, is attached to the ends of two identical elastic springs. The free ends of the springs are attached to two points A and B. The point C lies between A and B such that ABC is a straight line and $AC \neq BC$. The particle is held at C and then released from rest.

At time t seconds, the displacement of the particle from C is x m and its velocity is v ms^{-1}. The subsequent motion of the particle can be described by the differential equation $\ddot{x} = -25x$.

(a) Describe the motion of the particle.

Given that $x = 0.4$ and $v = 0$ when $t = 0$,

(b) solve the differential equation to find x as a function of t

(c) state the period of the motion and calculate the maximum speed of P.