8.2) Using trigonometric identities

Worked example

A curve has parametric equations

$$x = \sin t - 2, y = \cos t + 3, \quad t \in \mathbb{R}$$

Find:

a) A Cartesian equation of the curve in the form $y = f(x)$

b) Sketch the curve

Your turn

A curve has parametric equations

$$x = \sin t + 2, y = \cos t - 3, \quad t \in \mathbb{R}$$

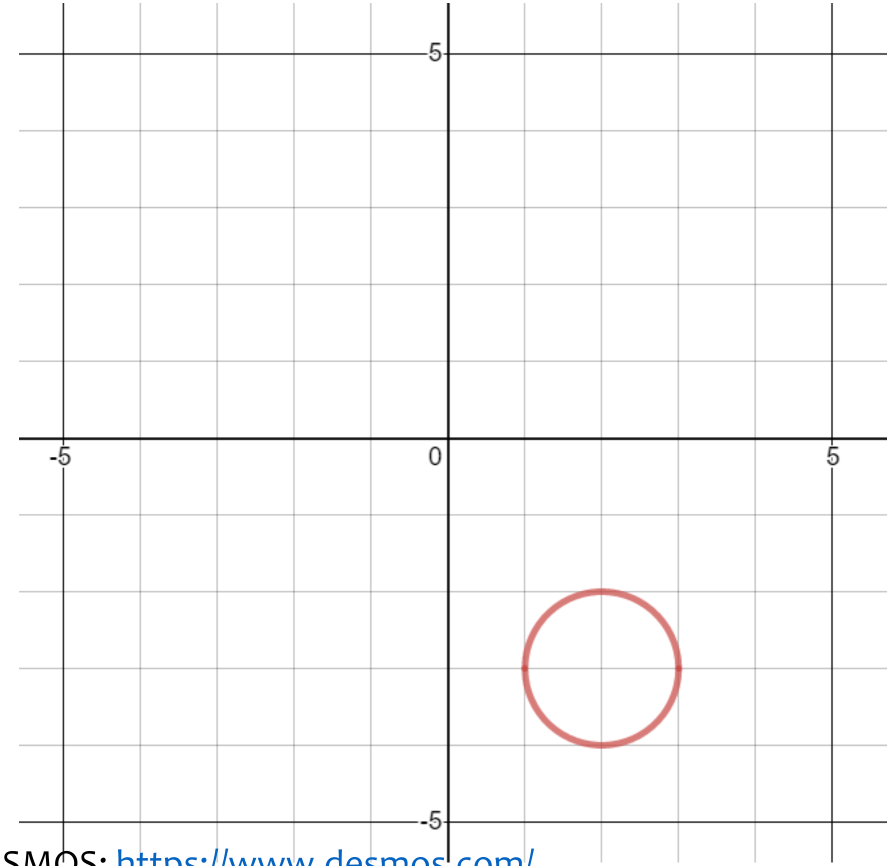
Find:

a) A Cartesian equation of the curve in the form $y = f(x)$

b) Sketch the curve

a) $(x-2)^2 + (y+3)^2 = 1$

b) Circle, radius 1, centre $(2, -3)$



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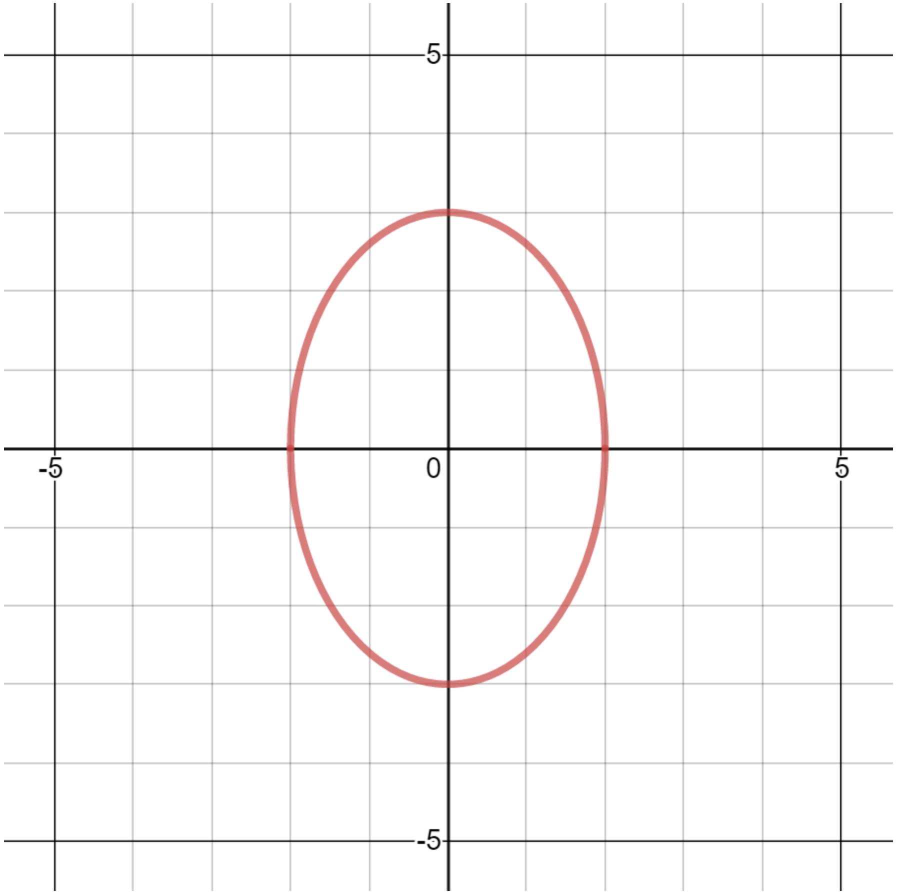
Worked example	Your turn
A curve has parametric equations $x = 2\sin t,\ y = 3\cos t,\quad t \in \mathbb{R}$ Find a Cartesian equation of the curve in the form $y = f(x)$	A curve has parametric equations $x = 3\sin t,\ y = 2\cos t,\quad t \in \mathbb{R}$ Find a Cartesian equation of the curve in the form $y = f(x)$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$



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Worked example

A curve has parametric equations

$$x = \cos t\,, \qquad y = \sin 2t\,, \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

Find:

a) A Cartesian equation of the curve in the form $y = f(x)$

b) The valid domain and range of $f(x)$

Your turn

A curve has parametric equations

$$x = \sin t\,, \qquad y = \sin 2t\,, \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

Find:

a) A Cartesian equation of the curve in the form $y = f(x)$

b) The valid domain and range of $f(x)$

a) $y = 2x\sqrt{1 - x^2}$

b) Domain: $-1 \le x \le 1$

Range: $-1 \le f(x) \le 1$

Worked example

A curve has parametric equations

$$x = 4\cos t\,, \qquad y = \cos 2t - 1, \qquad 0 \leq t \leq \pi$$

Find a Cartesian equation of the curve in the form

$y = f(x), -k \leq x \leq k$, stating the value of the constant k

Your turn

A curve has parametric equations

$$x = 2\sin t\,, \qquad y = 1 - \cos 2t\,, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

Find a Cartesian equation of the curve in the form

$y = f(x), -k \leq x \leq k$, stating the value of the constant k

$$y = \frac{x^2}{2}, -2 \leq x \leq 2\ (k = 2)$$

Worked example	Your turn
A curve has parametric equations $x = \cot t + 1, \quad y = cosec^2 t - 3, \quad 0 < t < \pi$ Find a Cartesian equation of the curve in the form $y = f(x)$ and state the domain of x for which the curve is defined	A curve has parametric equations $x = \cot t + 2, \quad y = cosec^2 t - 2, \quad 0 < t < \pi$ Find a Cartesian equation of the curve in the form $y = f(x)$ and state the domain of x for which the curve is defined $y = x^2 - 4x + 3, x \in \mathbb{R}$

Worked example

A curve has parametric equations

$$x = \sqrt{5}\sin 2t\,, \qquad y = 10\sin^2 t\,, \qquad 0 \le t < \pi$$

Find a Cartesian equation of the curve

Your turn

A curve has parametric equations

$$x = \sqrt{3}\sin 2t\,, \qquad y = 4\cos^2 t\,, \qquad 0 \le t < \pi$$

Find a Cartesian equation of the curve

$$x^2 = 3y\left(1 - \frac{y}{4}\right)$$

Worked example	Your turn
A curve has parametric equations $$x = 2\sin t\,, \qquad y = \sin\left(t + \frac{\pi}{6}\right), \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$ Find a Cartesian equation of the curve in the form $y = f(x)$ and state the domain of x for which the curve is defined	A curve has parametric equations $$x = 2\cos t\,, \qquad y = \sin\left(t - \frac{\pi}{6}\right), \qquad 0 < t < \pi$$ Find a Cartesian equation of the curve in the form $y = f(x)$ and state the domain of x for which the curve is defined $$y = \frac{1}{4}\left(\sqrt{12 - 3x^2} - x\right), -2 < x < 2$$

Worked example

A curve has parametric equations

$$x = \tan t\,, \qquad y = 5\sin(t - \pi)\,, \qquad 0 < t < \frac{\pi}{2}$$

Find a Cartesian equation of the curve

Your turn

A curve has parametric equations

$$x = \tan t\,, \qquad y = 4\sin(t + \pi)\,, \qquad 0 < t < \frac{\pi}{2}$$

Find a Cartesian equation of the curve

$$x = -\frac{y}{\sqrt{16 - y^2}}$$