8A Modelling with First Order Differentials

1. A particle P starts from rest at a given point O and moves along a straight line. At time t seconds, the acceleration, $a\ ms^{-2}$, of P is given by:

$$a = \frac{6}{(t-2)^2}, t \geq 0$$

a) Find the velocity of P at time t seconds

b) Show that the displacement of P from O when $t = 6$ is given by $(18 - 12ln2)\ m$

2. A particle P is travelling along a straight line. At time t seconds, the acceleration of the particle is given by:

$$a = t + \frac{3}{t}v,\ t \geq 0$$

Given that $v = 0$ when $t = 2$, show that the velocity of the particle at time t is given by the equation:

$$v = ct^3 - t^2$$

where c is a constant to be found.

3. A storage tank initially contains 1000 litres of pure water. Liquid is removed from the tank at a constant rate of 30 litres per hour and a chemical solution is added at a constant rate of 40 litres per hour. The chemical solution contains 4 grams of copper sulphate per litre of water.

a) Given that there are x grams of copper sulphate in the tank after t hours and that the copper sulphate immediately disperses throughout the tank upon entry, show that the situation can be modelled by the differential equation:

$$\frac{dx}{dt} = 160 - \frac{3x}{100 + t},\ t \geq 0$$

b) Hence, find the number of grams of copper sulphate in the tank after 6 hours.

c) Suggest a possible refinement for the model