CP2 Chapter 8

Modelling with Differential Equations

Course Structure

1. Modelling with 1st order differential equations.
2. Simple Harmonic Motion
3. Damped and Force Harmonic Motion
4. Coupled First-Order Differential Equations

Topic	What students need to learn:		
	Content		Guidance
9 Differential equations ***continued***	9.7	Solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion.	
	9.8	Model damped oscillations using second order differential equations and interpret their solutions.	Damped harmonic motion, with resistance varying as the derivative of the displacement, is expected. Problems may be set on forced vibration.
9 Differential equations *continued*	9.9	Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models.	Restricted to coupled first order linear equations of the form, $\frac{dx}{dt} = ax + by + f(t)$ $\frac{dy}{dt} = cx + dy + g(t)$

Modelling with 1st Order Differential Equations

Example

A particle P is moving along a straight line. At time t seconds, the acceleration of the particle is given by $a = t + \frac{3}{t}v, \ \ t \geq 0$

Given that $v = 0$ when $t = 2$, show that the velocity of the particle at time t is given by the equation $v = ct^3 - t^2$ where c is a constant to be found.

Common Example Type:

A storage tank initially containers 1000 litres of pure water. Liquid is removed from the tank at a constant rate of 30 litres per hour and a chemical solution is added to the tank at a constant rate of 40 litres per hour. The chemical solution contains 4 grams of copper sulphate per litre of water. Given that there are x grams of copper sulphate in the tank after t hours and that the copper sulphate immediately disperses throughout the tank on entry,

(a) Show that the situation can be modelled by the differential equation

$$\frac{dx}{dt} = 160 - \frac{3x}{100 + t}$$

(b) Hence find the number of grams of copper sulphate in the tank after 6 hours.

(c) Explain how the model could be refined.