**8A Modelling with First Order Differentials**

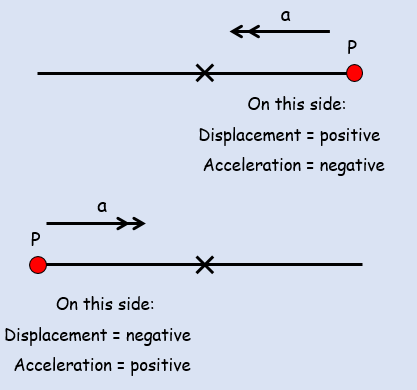
1. A particle starts from rest at a given point and moves along a straight line. At time seconds, the acceleration, , of is given by:
2. Find the velocity of at time seconds
3. Show that the displacement of from when is given by
4. A particle is travelling along a straight line. At time t seconds, the acceleration of the particle is given by:

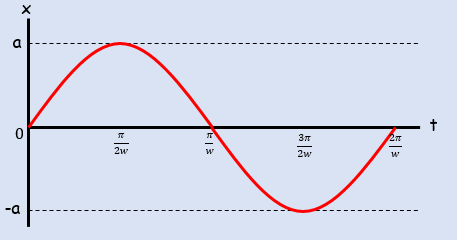
Given that when , show that the velocity of the particle at time t is given by the equation:

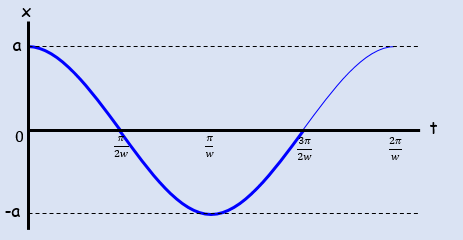
where is a constant to be found.

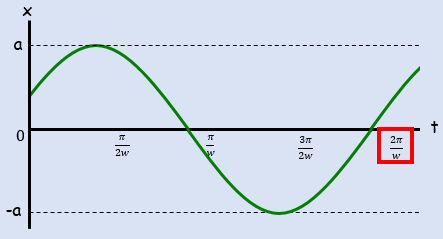
1. A storage tank initially contains 1000 litres of pure water. Liquid is removed from the tank at a constant rate of 30 litres per hour and a chemical solution is added at a constant rate of 40 litres per hour. The chemical solution contains 4 grams of copper sulphate per litre of water.
2. Given that there are grams of copper sulphate in the tank after hours and that the copper sulphate immediately disperses throughout the tank upon entry, show that the situation can be modelled by the differential equation:
3. Hence, find the number of grams of copper sulphate in the tank after 6 hours.
4. Suggest a possible refinement for the model

**8B Simple Harmonic Motion**









1. A particle is moving along a straight line. At time seconds its displacement, from a fixed point is such that:

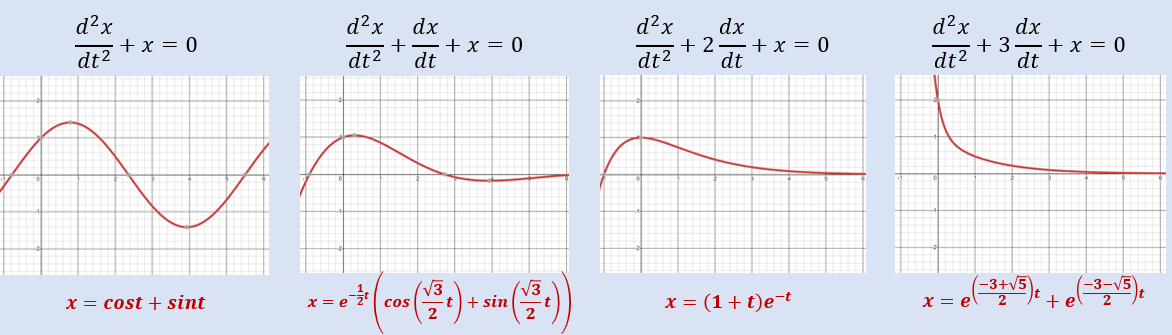
Given that at , and that the particle is moving with velocity :

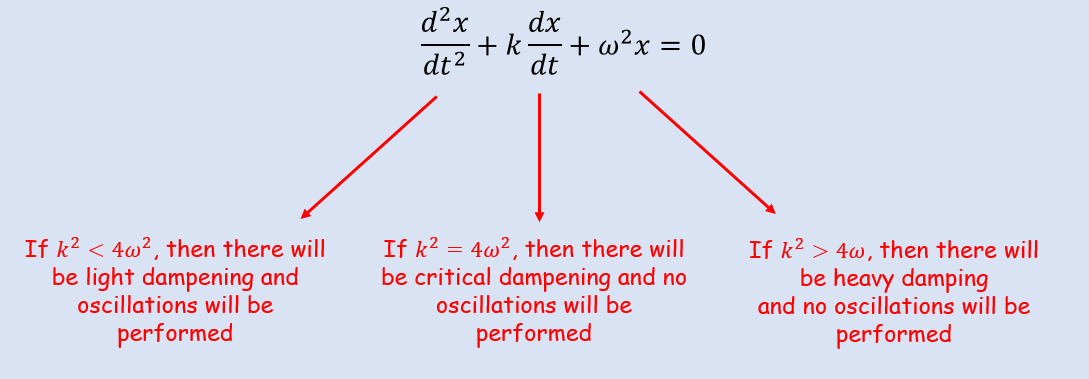
1. Find an expression for the particle’s displacement after seconds
2. Determine the maximum displacement of the particle from .
3. A particle is attached to the ends of two identical elastic springs. The free ends of the springs are attached to two points and . The point lies between and such that is a straight line and . The particle is held at and then released from rest.

At time seconds, the displacement of the particle from is and its velocity is . The subsequent motion can be described by the differential equation .

1. Describe the motion of the particle
2. Given that when , and , find as a function of
3. State the period of the motion and state the maximum speed of .

**8C Part 1 Damped Harmonic Motion**





1. A particle of mass 0.5kg moves in a horizontal straight line. At time seconds, the displacement of from a fixed point , on the line is and the velocity of is . A force of magnitude acts on in the direction . The particle is also subject to a resistance of magnitude . When , and is moving in the direction of increasing with speed .
2. Show that
3. Find the value of when
4. A particle hangs freely in equilibrium attached to one end of a light elastic string. The other end of the string is attached to a fixed point . The particle is pulled down and held at rest in a container of liquid which exerts a resistance on the motion on . is then released from rest. While the string remains taut and the particle in the liquid, the motion can be modelled using the equation:

Where is a positive real constant.

Find the general solution to the differential equation and state the type of damping the particle is subject to.

1. One end of a light elastic spring is attached to a fixed point . A particle is attached to the other end and hangs in equilibrium vertically below . The particle is pulled vertically down from its equilibrium position and released from rest. A resistance proportional to the speed of acts on .

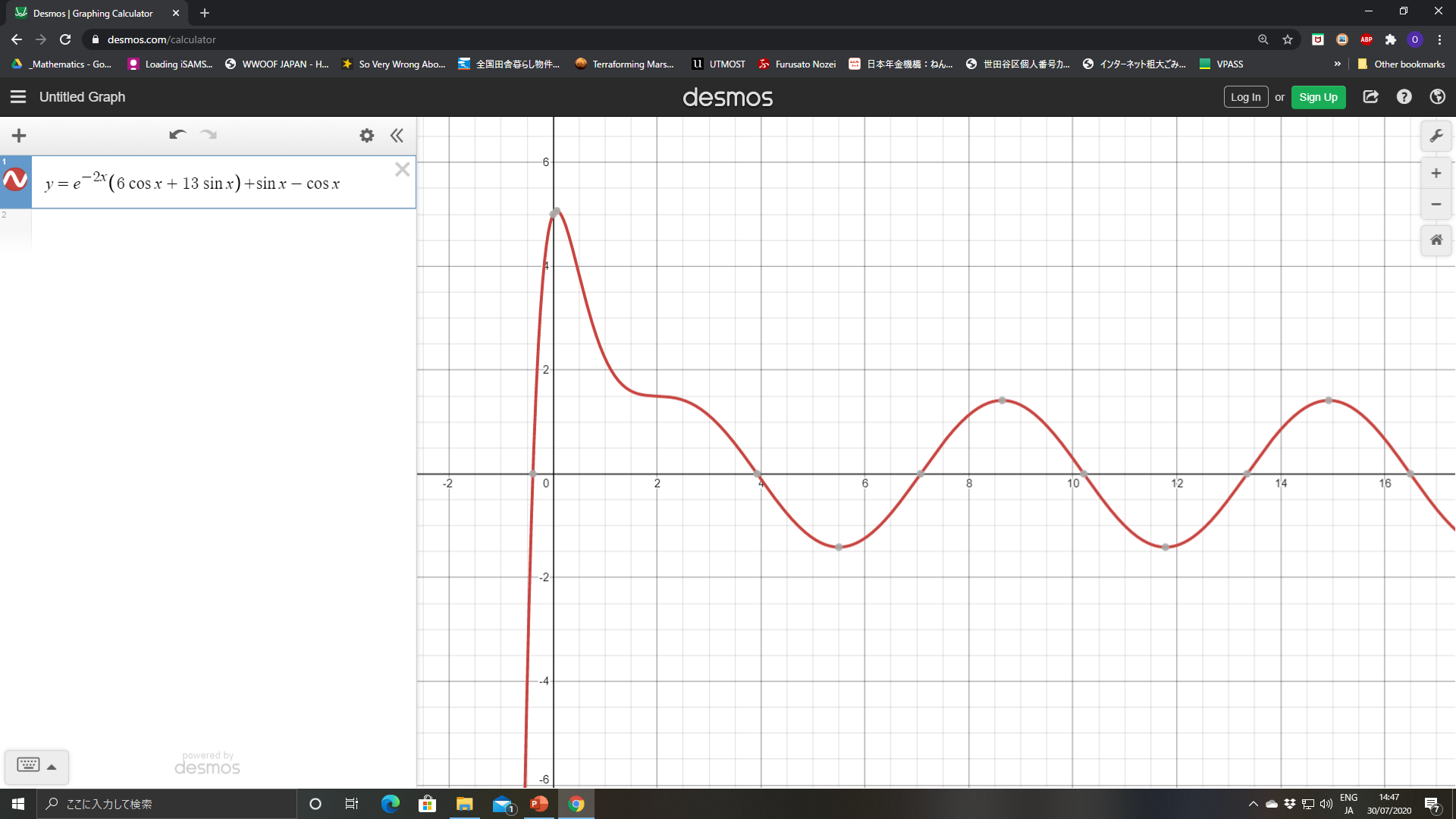
The equation of motion of is given as:

Where is a positive real constant and is the displacement of from its equilibrium position.

1. Find the general solution to the differential equation.
2. Find the period of the motion

**8C Part 2 Forced Harmonic Motion**

1. A particle of mass 1.5kg is moving along the x-axis. At time the displacement of from the origin is metres and the speed of is . Three forces act on , namely a restoring force of , a resistance to motion of of magnitude and a force of magnitude acting in the direction . When , and .
2. Show that
3. Find as a function of
4. Describe the motion when is large



1. A particle is attached to end of a light elastic string . Initially the particle and the string lie at rest on a smooth horizontal plane. At time , the end of the string is set into motion and moves with constant speed in the direction , and the extension in the string is . Air resistance acting on is proportional to its speed. The subsequent motion can be modelled by the differential equation:

Find an expression for in terms of , and .

**8D Coupled Differential Equations**

1. At the start of 2010, a survey began on the number of bears and fish on a remote island in Northern Canada. After years the number of bears, , and the number of fish, , in the area are modelled by the differential equations:
2. Show that
3. Find the general solution for the number of bears on the island at time
4. Find the general solution for the number of fish on the island at time
5. At the start of 2010, there were 5 bears and 20 fish on the island. Use this information to predict the number of bears on the island in 2020.
6. Comment on the suitability of the model

no other factors were taken into account as well – the f(t) and g(t) mentioned at the start

1. Two barrels contain contaminated water. At time seconds, the amount of contaminant in barrel is ml and the amount in barrel is ml. Additional contaminated water flows into barrel at a rate of 5 ml per second. Contaminated water flows from barrel to barrel and then back to barrel through two connecting hoses, and then drains out of barrel to leave the system completely. The system is modelled using the following differential equations:

Show that