8A Movement on a Plane

1. A particle starts from the point with position vector $(3\boldsymbol{i} + 7\boldsymbol{j})m$ and moves with constant velocity $(2\boldsymbol{i} - \boldsymbol{j})\ ms^{-1}$.

a) Find the position vector of the particle after 4 seconds

b) Find the time at which the particle is due east of the origin

2. A particle P has velocity $(-3\boldsymbol{i} + \boldsymbol{j})\ ms^{-1}$ at time $t = 0$. The particle moves with constant acceleration $a = (2\boldsymbol{i} + 3\boldsymbol{j})\ ms^{-2}$. Find the speed of the particle and the bearing on which it is travelling at time 3 seconds.

3. An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is skating with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})\ ms^{-1}$.

   At time $t = 20$ the skater is travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})\ ms^{-1}$.

   Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds.

   Modelling the skater as having constant acceleration, find:

a) The acceleration of the ice skater

b) An expression for $\boldsymbol{s}$ in terms of $\boldsymbol{t}$

c) Find the time at which the skater is directly north-east of O

d) A second skater travels such that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}\ m$ relative to the same point O at time t.

8B Projectiles with Vectors

1. A ball is struck by a racket from a point A which has position vector $20\boldsymbol{j}\ m$ relative to a fixed origin O. Immediately after being struck, the ball has velocity $(5\boldsymbol{i} + 8\boldsymbol{j})\ ms^{-1}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point B.

a) Find the speed of the ball 1.5 seconds after being struck

b) Find an expression for the position vector, $\boldsymbol{r}$ of the ball relative to O at time t seconds

c) Hence determine the distance OB

8C Calculus in Mechanics

1. A particle is moving in a straight line with acceleration at time t seconds given by:

   $a = cos2\pi t\ ms^{-2}, \quad t \geq 0$

   The velocity of the particle at time $t = 0$ is $\frac{1}{2\pi}\ ms^{-1}$. Find:

a) An expression for the velocity at time t seconds

b) The maximum speed of the particle

c) The distance travelled in the first 3 seconds

2. A particle of mass 6kg is moving on the positive x-axis. At time t seconds the displacement, s, of the particle from the origin is given by:

   $s = \left(2t^{\frac{3}{2}} + \frac{e^{-2t}}{3}\right)m$, where $t \geq 0$

a) Find the velocity of the particle when $t = 1.5$

b) Given that the particle is acted on by a single force of variable magnitude $F\ N$ which acts in the direction of the positive x-axis, find the value of F when $t = 2$

8D Differentiating Vectors

1. A particle P of mass 0.8kg is acted on by a single force $\boldsymbol{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where:

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \quad t \geq 0$$

Find:

a) The speed of P when $t = 4$

b) The acceleration of P as a vector when $t = 2$

c) The value of $\boldsymbol{F}$ when $t = 2$

8E Integrating Vectors

1. A particle P is moving in a plane. At time t seconds, its velocity, $\boldsymbol{v} ms^{-1}$, is given by:

$$v = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}$$

When $t = 0$, the position vector of P with respect to a fixed origin O is $(2\boldsymbol{i} - 3\boldsymbol{j})\ m$. Find the position vector of P at time t seconds

2. A particle P is moving in a plane so that, at time t seconds, its acceleration is:

$$\boldsymbol{a} = (4\boldsymbol{i} - 2t\boldsymbol{j})ms^{-2}$$

At $t = 3$, the velocity of P is $6\boldsymbol{i}\ ms^{-1}$ and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})\ m$ with respect to a fixed origin O. Find:

a) The angle between the direction of motion of P, and $\boldsymbol{i}$, when $t = 2$

b) The distance of P from O when $t = 0$

3. The velocity of a particle at time t seconds is given by:

$$\boldsymbol{v} = (3t^2 - 8)\boldsymbol{i} + 5\boldsymbol{j}$$

When $t = 0$, the position vector of P with respect to a fixed origin is $(2\boldsymbol{i} - 4\boldsymbol{j})\ m$

a) Find the position vector of P after t seconds

A second particle Q moves with constant velocity $(8\boldsymbol{i} + 4\boldsymbol{j})\ ms^{-1}$. When $t = 0$, the position vector of Q with respect to the origin O is $2\boldsymbol{i}\ m$.

b) Prove that P and Q collide