

# 8) The binomial expansion

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# 8.1) Pascal's triangle

## Worked example

Use Pascal's triangle to find the expansion of  
 $(x + 4y)^5$

## Your turn

Use Pascal's triangle to find the expansion of  
 $(x + 2y)^3$

$$x^3 + 6x^2y + 12xy^2 + 8y^3$$

## Worked example

Use Pascal's triangle to find the expansion of  
 $(5x - 2)^4$

## Your turn

Use Pascal's triangle to find the expansion of  
 $(2x - 5)^4$

$$16x^4 - 160x^3 + 600x^2 - 1000x + 625$$

## Worked example

Find the expansion of  $(3 + 2x)^4$

## Your turn

Find the expansion of  $(2 + 3x)^4$

$$16 + 96x + 216x^2 + 216x^3 + 81x^4$$

## Worked example

Find the expansion of  $(1 - 4x)^3$

## Your turn

Find the expansion of  $(1 - 2x)^3$

$$1 - 6x + 12x^2 - 8x^3$$

## Worked example

The coefficient of  $x^2$  in the expansion of  $(2 - cx)^3$  is 294.

Find the possible value(s) of the constant  $c$ .

## Your turn

The coefficient of  $x^2$  in the expansion of  $(2 - cx)^5$  is 720.

Find the possible value(s) of the constant  $c$ .

$$c = \pm 3$$

## Worked example

The coefficient of  $x^3$  in the expansion of  $(3 - cx)^5$  is 720.

Find the possible value(s) of the constant  $c$ .

## Your turn

The coefficient of  $x^3$  in the expansion of  $(2 - cx)^5$  is  $-1080$ .

Find the possible value(s) of the constant  $c$ .

$$c = 3$$



## Worked example

In the binomial expansion of  $(3 + kx)^7$ , where  $k$  is a constant, the coefficient of  $x^2$  is 2 times the coefficient of  $x$ .

Find the value of  $k$

## Your turn

In the binomial expansion of  $(2 + kx)^7$ , where  $k$  is a constant, the coefficient of  $x^2$  is 6 times the coefficient of  $x$ .

Find the value of  $k$

$$k = 4$$

## 8.2) Factorial notation

## Worked example

Find the number of different ways of arranging the letters  $ABCD$

## Your turn

Find the number of different ways of arranging the letters  $ABCDE$

$$5! = 120$$

## Worked example

Find the number of ways of a football coach choosing 11 starting players from a squad of 18

## Your turn

Find the number of ways of a netball coach choosing 7 starting players from a squad of 12

$$\frac{12!}{7!5!} = 792$$

## Worked example

Using factorials, evaluate:

$$1!$$

$$\binom{10}{0}$$

## Your turn

Using factorials, evaluate:

$$0!$$

$$1$$

$$\binom{20}{1}$$

$$20$$

$$190$$

$$190$$

## 8.3) The binomial expansion

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## Worked example

Use the binomial theorem to find the expansion of  $(2 - 3x)^5$

## Your turn

Use the binomial theorem to find the expansion of  $(3 - 2x)^5$

$$243 - 810x + 1080x^2 - 720x^3 + 240x^4 - 32x^5$$

## Worked example

Find the first four terms in ascending powers of  $x$  in the binomial expansion of  $(1 + 3x)^{11}$

## Your turn

Find the first four terms in ascending powers of  $x$  in the binomial expansion of  $(1 + 2x)^{10}$

$$1 + 20x + 180x^2 + 960x^3 + \dots$$



## Worked example

Find the first four terms in ascending powers of  $x$  in the binomial expansion of

$$\left(6 - \frac{1}{3}x\right)^{10}$$

## Your turn

Find the first four terms in ascending powers of  $x$  in the binomial expansion of

$$\left(10 - \frac{1}{2}x\right)^6$$

$$1000000 - 300000x + 37500x^2 - 2500x^3 + \dots$$

## Worked example

Find the first 3 terms in the expansion of  $\left(3 - \frac{1}{2}x\right)^5$ , in ascending powers of  $x$ .

## Your turn

Find the first 3 terms in the expansion of  $\left(2 - \frac{1}{3}x\right)^7$ , in ascending powers of  $x$ .

$$128 - \frac{448}{3}x + \frac{224}{3}x^2 + \dots$$

## Worked example

Find the binomial expansion of  $\left(x + \frac{1}{x}\right)^7$  giving each term in its simplest form

## Your turn

Find the binomial expansion of  $\left(x + \frac{1}{x}\right)^5$  giving each term in its simplest form

$$x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$$

## 8.4) Solving binomial problems

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## Worked example

Find the coefficient of  $x^6$  in the binomial expansion of  $(3 + 2x)^{10}$

## Your turn

Find the coefficient of  $x^4$  in the binomial expansion of  $(2 + 3x)^{10}$

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## Worked example

Find the coefficient of  $x^3$  in the binomial expansion of  $(3 + x)(2 - 3x)^7$

## Your turn

Find the coefficient of  $x^3$  in the binomial expansion of  $(2 + x)(3 - 2x)^7$

**-24948**

## Worked example

The coefficient of  $x^6$  in the expansion of  $(1 + qx)^{10}$  is 153090. Find the possible value(s) of the constant  $q$ .

## Your turn

The coefficient of  $x^4$  in the expansion of  $(1 + qx)^{10}$  is 3360. Find the possible value(s) of the constant  $q$ .

$$q = \pm 2$$

## Worked example

In the expansion of  $(1 + ax)^8$ , where  $a$  is a non-zero constant the coefficient of  $x^3$  is quadruple the coefficient of  $x^2$ .

Find the value of  $a$ .

## Your turn

In the expansion of  $(1 + ax)^{10}$ , where  $a$  is a non-zero constant the coefficient of  $x^3$  is double the coefficient of  $x^2$ .

Find the value of  $a$ .

$$a = \frac{3}{4}$$



## Worked example

Given that, in the expansion of  $(1 + qx)^8$ , the coefficient of  $x$  is  $-r$  and the coefficient of  $x^2$  is  $14r$ , find the value of  $q$  and the value of  $r$

## Your turn

Given that, in the expansion of  $(1 + qx)^8$ , the coefficient of  $x$  is  $-r$  and the coefficient of  $x^2$  is  $7r$ , find the value of  $q$  and the value of  $r$

$$q = -2, r = 16$$

## Worked example

In the binomial expansion of  $(1 + x)^{40}$ , the coefficients of  $x^{19}$  and  $x^{20}$  are  $p$  and  $q$  respectively. Find the value of  $\frac{q}{p}$

## Your turn

In the binomial expansion of  $(1 + x)^{20}$ , the coefficients of  $x^9$  and  $x^{11}$  are  $p$  and  $q$  respectively. Find the value of  $\frac{q}{p}$

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## 8.5) Binomial estimation

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## Worked example

a) Find the first four terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 + \frac{x}{2}\right)^{10}$$

b) Use your expansion to estimate the value of  $1.052^{10}$ , giving your answer to 4 decimal places

## Your turn

a) Find the first four terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 + \frac{x}{4}\right)^8$$

b) Use your expansion to estimate the value of  $1.025^8$ , giving your answer to 4 decimal places

a)  $1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \dots$

b)  $1.2184$  (4 dp)

## Worked example

a) Find the first four terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 - \frac{x}{2}\right)^8$$

b) Use your expansion to estimate the value of  $0.957^8$ , giving your answer to 4 decimal places

## Your turn

a) Find the first four terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 - \frac{x}{4}\right)^{10}$$

b) Use your expansion to estimate the value of  $0.975^{10}$ , giving your answer to 4 decimal places

a)  $1 - \frac{5}{2}x + \frac{45}{16}x^2 - \frac{15}{8}x^3 + \dots$

b)  $0.7763$  (4 dp)

## Worked example

a) Find the first three terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(5 - \frac{x}{7}\right)^6$$

b) Use your expansion to estimate the value of  $4.996^9$ , giving your answer to 4 significant figures

## Your turn

a) Find the first three terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(7 - \frac{x}{5}\right)^9$$

b) Use your expansion to estimate the value of  $6.991^8$ , giving your answer to 4 significant figures

a)  $40353607 - \frac{51883209}{5}x + \frac{29647548}{25}x^2 + \dots$

b)  $39890000$

## Worked example

a) Find the first three terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 - \frac{x}{3}\right)^8$$

b) Use your expansion to estimate the value of  $0.96^8$ , giving your answer to 5 decimal places

## Your turn

a) Find the first four terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 - \frac{x}{4}\right)^8$$

b) Use your expansion to estimate the value of  $0.96^8$ , giving your answer to 5 decimal places

a)  $1 - 2x + \frac{7}{4}x^2 - \frac{7}{8}x^3 + \dots$

b)  $0.72122$