8) Modelling in Mechanics

8.1) Constructing a model

Worked example	Your turn
A stone is thrown from the top of a cliff into the sea. The height of the stone above sea level, h m, at time t seconds after it is thrown can be modelled using the equation $h = -5t^2 + 15t + 90$	A basketball is thrown into a net. The height of the basketball above the ground can be modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where x m is the horizontal distance travelled.
a) Find the height of the stone above sea level: i) When it is released ii) 8 seconds after it is thrown b) Use the model to predict the height of the stone above sea level after 20 seconds. c) Comment on the validity of this prediction. d) The model is only valid from the time the stone is thrown until the time it enters the sea. Find the range of values of t for which the model is valid.	a) Find the height of the basketball: i) When it is released ii) At a horizontal distance of 0.5 m b) Use the model to predict the height of the basketball when it is at a horizontal distance of 15 m from the player. c) Comment on the validity of this prediction. d) The model is only valid when the balls is above the ground. Find the range of values of x for which the model is valid.
	a) i) 2 m ii) 2.525 m b) -4 m c) Height cannot be negative, so the model is not valid when $x = 15$ m d) $0.00 \leq x < 12.59$ (2 dp)

8.2) Modelling assumptions

Worked example	Your turn
List assumptions you would make to create a simple model of: • The motion of two objects of different masses connected by a string that passes over a pulley	List assumptions you would make to create a simple model of: • The motion of a golf ball after it is hit Model the golf ball as a particle. Ignore the effects of air resistance. Ignore the rotational effect of any external forces acting on it.
• The motion of a child on a sledge going down a snow-covered hill	• The motion of a suitcase on wheels being pulled along a path by its handle Model the suitcase and handle as a single particle, consider the path to be smooth, and ignore friction between the wheels and their holdings.

8.3) Quantities and units

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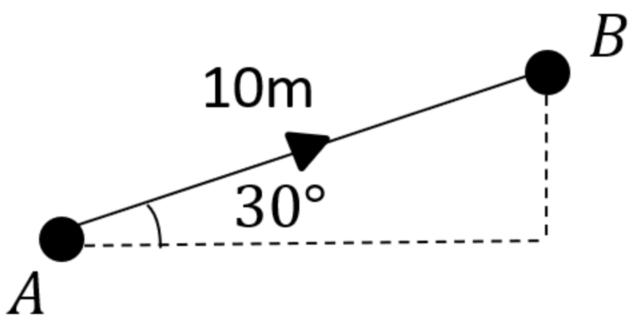
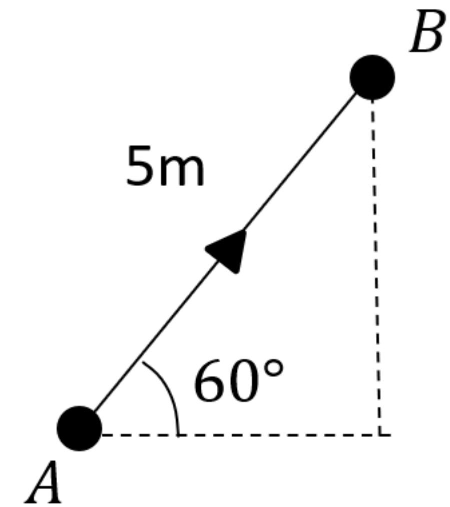
Worked example	Your turn
Convert to SI units:	Convert to SI units:
• $56\ km\ h^{-1}$	• $65\ km\ h^{-1}$ $18.1\ ms^{-1}$ (3 sf)
• $51\ g\ cm^{-2}$	• $15\ g\ cm^{-2}$ $150\ kg\ m^{-2}$
• $40\ cm$ per minute	• $30\ cm$ per minute $5 \times 10^{-3}\ ms^{-1}$
• $42\ g\ m^{-3}$	• $24\ g\ m^{-3}$ $2.4 \times 10^{-2}\ kg\ m^{-3}$
• $5.4 \times 10^{-3}\ g\ cm^{-3}$	• $4.5 \times 10^{-2}\ g\ cm^{-3}$ $45\ kg\ m^{-3}$
• $3.6 \times 10^{-2}\ kg\ cm^{-2}$	• $6.3 \times 10^{-3}\ kg\ cm^{-2}$ $63\ kg\ m^{-2}$

8.4) Working with vectors

Worked example	Your turn
Convert from scalar to vector form:	Convert from scalar to vector form:

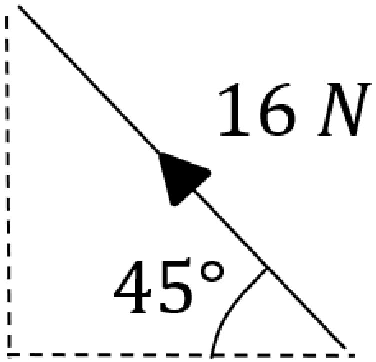


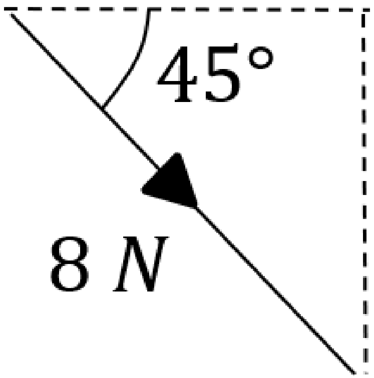


Displacement (from A) $= \begin{pmatrix} 5\cos 60° \\ 5\sin 60° \end{pmatrix} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix} m$

Diagrams/Graphs used with permission from DrFrostMaths: https://www.drfrostmaths.com/

Worked example	Your turn
Convert from scalar to vector form:	Convert from scalar to vector form:





Force vector $= \begin{pmatrix} 8cos45° \\ -8\sin 45° \end{pmatrix} = \begin{pmatrix} 4\sqrt{2} \\ -4\sqrt{2} \end{pmatrix} N$

Worked example

Convert from vector to scalar form:

$$\text{Velocity} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} ms^{-1}$$

Your turn

Convert from vector to scalar form:

$$\text{Velocity} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} ms^{-1}$$

$$\text{Speed} = 13\ ms^{-1}$$

Worked example	Your turn
Convert from vector to scalar form: Acceleration $= (3\boldsymbol{i} - 4\boldsymbol{j})\ ms^{-2}$	Convert from vector to scalar form: Acceleration $= (-6\boldsymbol{i} + 8\boldsymbol{j})\ ms^{-2}$ Magnitude of the acceleration $= 10\ ms^{-1}$

Worked example	Your turn
The velocity of a particle is given by $v = 2\boldsymbol{i} + 7\boldsymbol{j}\ ms^{-1}$. Find: a) The speed of the particle b) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{i}$ c) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{j}$	The velocity of a particle is given by $v = 3\boldsymbol{i} + 5\boldsymbol{j}\ ms^{-1}$. Find: a) The speed of the particle b) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{i}$ c) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{j}$ a) 5.83 ms^{-1} (2 dp) b) 59.04° (2 dp) c) 30.96° (2 dp)

Worked example

The velocity of a particle is given by

$$v = 3\boldsymbol{i} - 5\boldsymbol{j}\ ms^{-1}.$$

Find:

a) The speed of the particle
b) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{i}$
c) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{j}$

Your turn

The velocity of a particle is given by

$$v = 2\boldsymbol{i} - 7\boldsymbol{j}\ ms^{-1}.$$

Find:

a) The speed of the particle
b) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{i}$
c) The angle the direction of motion of the particle makes with the unit vector $\boldsymbol{j}$

a) $7.28\ ms^{-1}$ (2 dp)
b) $74.05°$ (2 dp)
c) $164.05°$ (2 dp)

Worked example	Your turn
A man walks from A to B and then from B to C. His displacement from A to B is $5\boldsymbol{i} - 6\boldsymbol{j}$ m. His displacement from B to C is $4\boldsymbol{i} + 12\boldsymbol{j}$ m. a) What is the magnitude of the displacement from A to C? b) What is the total distance the man has walked in getting from A to C.	A man walks from A to B and then from B to C. His displacement from A to B is $6\boldsymbol{i} + 4\boldsymbol{j}$ m. His displacement from B to C is $5\boldsymbol{i} - 12\boldsymbol{j}$ m. a) What is the magnitude of the displacement from A to C? b) What is the total distance the man has walked in getting from A to C. a) 13.60 km (2 dp) b) 20.21 km (2 dp)