8) Further kinematics

8.1) Vectors in kinematics

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Worked example

A particle starts from the position vector $(7\boldsymbol{i} - 2\boldsymbol{j})$ m and moves with constant velocity $(-3\boldsymbol{i} + \boldsymbol{j})$ ms^{-1}.

(a) Find the position vector of the particle 2 seconds later.

(b) Find the time at which the particle is due north of the origin.

Your turn

A particle starts from the position vector $(3\boldsymbol{i} + 7\boldsymbol{j})$ m and moves with constant velocity $(2\boldsymbol{i} - \boldsymbol{j})$ ms^{-1}.

(a) Find the position vector of the particle 4 seconds later.

(b) Find the time at which the particle is due east of the origin.

a) $(11\boldsymbol{i} + 3\boldsymbol{j})\ ms^{-1}$

b) 7 s

Worked example	Your turn
A particle P has velocity $(-\boldsymbol{i} + 5\boldsymbol{j})$ ms^{-1}. The particle moves with constant acceleration $\boldsymbol{a} = (4\boldsymbol{i} + 7\boldsymbol{j})$ ms^{-2}. Find: (a) the speed of the particle at time $t = 6$ seconds. (b) the bearing on which it is travelling at time $t = 6$ seconds.	A particle P has velocity $(-3\boldsymbol{i} + \boldsymbol{j})$ ms^{-1}. The particle moves with constant acceleration $\boldsymbol{a} = (2\boldsymbol{i} + 3\boldsymbol{j})$ ms^{-2}. Find: (a) the speed of the particle at time $t = 3$ seconds. (b) the bearing on which it is travelling at time $t = 3$ seconds. a) $10.4\ ms^{-1}$ (3 sf) b) $017°$

Worked example	Your turn
An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is travelling with velocity $(-4\boldsymbol{i} - 9\boldsymbol{j})\ ms^{-1}$. At time $t = 5$ s the skater is travelling with velocity $(-34\boldsymbol{i} + 29\boldsymbol{j})\ ms^{-1}$. Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds. Modelling the ice skater as a particle with constant acceleration, find: (a) The acceleration of the ice skater (b) An expression for $\boldsymbol{s}$ in terms of t (c) The time at which the skater is directly south-west of O. A second skater travels so that she has position vector $\boldsymbol{r} = (-132\boldsymbol{i} + (6 - 22t)\boldsymbol{j})$ m relative to O at time t. (d) Show that the two skaters will meet.	An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is travelling with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})\ ms^{-1}$. At time $t = 20$ s the skater is travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})\ ms^{-1}$. Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds. Modelling the ice skater as a particle with constant acceleration, find: (a) The acceleration of the ice skater (b) An expression for $\boldsymbol{s}$ in terms of t (c) The time at which the skater is directly north-east of O. A second skater travels so that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}$ m relative to O at time t. (d) Show that the two skaters will meet. a) $(-0.4\boldsymbol{i} + 0.2\boldsymbol{j})\ ms^{-2}$ b) $\left((2.4t - 0.2t^2)\boldsymbol{i} + (-0.6t + 0.1t^2)\boldsymbol{j}\right)\ m$ c) $t = 10$ s d) Shown: Meet when $t = 12$ s

Worked example

A ship S is moving with constant velocity $(2\boldsymbol{i} + 4\boldsymbol{j})\ kmh^{-1}$.
At time $t = 0$, the position vector of S is $(-3\boldsymbol{i} + 5\boldsymbol{j})\ km$.
A ship T is moving with constant velocity $(6\boldsymbol{i} + n\boldsymbol{j})\ kmh^{-1}$
At time $t = 0$, the position vector of T is $(-15\boldsymbol{i} + 2\boldsymbol{j})\ km$.
The two ships meet at point P.
Find the value of n and the distance OP

Your turn

A ship S is moving with constant velocity $(3\boldsymbol{i} + 3\boldsymbol{j})\ kmh^{-1}$.
At time $t = 0$, the position vector of S is $(-4\boldsymbol{i} + 2\boldsymbol{j})\ km$.
A ship T is moving with constant velocity $(-2\boldsymbol{i} +$

$n = 3.5, OP = 8.25\ km$ (3 sf)

8.2) Vector methods with projectiles

Worked example	Your turn
A ball is struck by a racket from a point A which has position vector $40j$ m relative to a fixed origin O. Immediately after being struck, the ball has velocity $(7i + 10j)$ ms^{-1}, where i and j are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point B. (a) Find the speed of the ball 3 seconds after being struck. (b) Find an expression for the position vector, r, of the ball relative to O at time t seconds. (c) Hence determine the distance OB.	A ball is struck by a racket from a point A which has position vector $20j$ m relative to a fixed origin O. Immediately after being struck, the ball has velocity $(5i + 8j)$ ms^{-1}, where i and j are unit vectors horizontally and vertically respectively. After being struck, the ball travels freely under gravity until it strikes the ground at point B. (a) Find the speed of the ball 1.5 seconds after being struck. (b) Find an expression for the position vector, r, of the ball relative to O at time t seconds. (c) Hence determine the distance OB. a) $8.4\ ms^{-1}$ (2 sf) b) $\left((5t)\boldsymbol{i} + (8t - 4.9t^2 + 20)\boldsymbol{j}\right)\ m$ c) $15\ m$ (2 sf)

Worked example

The point O is a fixed point on a horizontal plane.
A ball is projected from O with velocity $(4\boldsymbol{i} + 8\boldsymbol{j})\ ms^{-1}$.
The ball passes through a point A at time t seconds after projection. The point B is on the horizontal plane vertically below A. It is given that $OB = 4AB$. Find:

a) The value of t

b) The speed of the ball at the instant it passes through A

Your turn

The point O is a fixed point on a horizontal plane.
A ball is projected from O with velocity $(6\boldsymbol{i} + 12\boldsymbol{j})\ ms^{-1}$.
The ball passes through a point A at time t seconds after projection. The point B is on the horizontal plane vertically below A. It is given that $OB = 2AB$. Find:

a) The value of t

b) The speed of the ball at the instant it passes through A

a) $t = 1.8$ (2 sf)

b) $8.5\ ms^{-1}$ (2 sf)

8.3) Variable acceleration in one dimension

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Worked example

A particle is moving in a straight line with acceleration at time t seconds given by

$$a = \cos 5\pi t \text{ ms}^{-2}, \qquad t \geq 0$$

The velocity of the particle at time $t = 0$ is $\frac{1}{5\pi} ms^{-1}$. Find:

(a) an expression for the velocity at time t seconds
(b) the maximum speed
(c) the distance travelled in the first 6 seconds.

Your turn

A particle is moving in a straight line with acceleration at time t seconds given by

$$a = \cos 2\pi t \text{ ms}^{-2}, \qquad t \geq 0$$

The velocity of the particle at time $t = 0$ is $\frac{1}{2\pi} ms^{-1}$. Find:

(a) an expression for the velocity at time t seconds
(b) the maximum speed
(c) the distance travelled in the first 3 seconds.

a) $v = \frac{1}{2\pi}\sin 2\pi t + \frac{1}{2\pi} ms^{-1}$

b) $\frac{1}{\pi} = 0.32\ ms^{-1}$ (2 sf)

c) $0.48\ m$ (2 sf)

Worked example	Your turn
A particle of mass 12kg is moving on the positive x-axis. At time t seconds the displacement, s, of the particle from the origin is given by $$s = 3t^{\frac{5}{2}} + \frac{e^{-3t}}{4}\ \mathrm{m}, \qquad t \geq 0$$ (a) Find the velocity of the particle when $t = 2.5$. Given that the particle is acted on by a single force of variable magnitude F N which acts in the direction of the positive x-axis, (b) Find the value of F when $t = 4$	A particle of mass 6kg is moving on the positive x-axis. At time t seconds the displacement, s, of the particle from the origin is given by $$s = 2t^{\frac{3}{2}} + \frac{e^{-2t}}{3}\ \mathrm{m}, \qquad t \geq 0$$ (a) Find the velocity of the particle when $t = 1.5$. Given that the particle is acted on by a single force of variable magnitude F N which acts in the direction of the positive x-axis, (b) Find the value of F when $t = 2$ a) 3.6 ms^{-1} (2 sf) b) 6.5 N (2 sf)

8.4) Differentiating vectors

Worked example	Your turn
A particle P of mass 1.6kg is acted on by a single force $\mathbf{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where	A particle P of mass 0.8kg is acted on by a single force $\mathbf{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where
$$\boldsymbol{r} = 5t^3\boldsymbol{i} + 20t^{-\frac{1}{5}}\boldsymbol{j}, \qquad t \geq 0$$	$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$
Find:	Find:
(a) the speed of P when $t = 2$	(a) the speed of P when $t = 4$
(b) the acceleration of P as a vector when $t = 4$	(b) the acceleration of P as a vector when $t = 2$
(c) $\mathbf{F}$ when $t = 4$.	(c) $\mathbf{F}$ when $t = 2$.
	a) 96 ms^{-1} (2 sf)
	b) $(24\boldsymbol{i} + 6.6\boldsymbol{j})$ ms^{-2} (2 sf)
	c) $(19\boldsymbol{i} + 5.3\boldsymbol{j})$ N (2 sf)

8.5) Integrating vectors

Worked example	Your turn
A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms⁻¹ is given by $$\boldsymbol{v} = 2t\boldsymbol{i} + \frac{1}{3}t^2\boldsymbol{j}, \qquad t \geq 0$$ When $t = 0$, the position vector of P with respect to a fixed O is $(5\boldsymbol{i} - 4\boldsymbol{j})$ m. Find the position vector of P at time t seconds.	A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms⁻¹ is given by $$\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}, \qquad t \geq 0$$ When $t = 0$, the position vector of P with respect to a fixed O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector of P at time t seconds. $$\left(\left(\frac{3t^2}{2} + 2\right)\boldsymbol{i} + \left(\frac{t^3}{6} - 3\right)\boldsymbol{j}\right) m$$

Worked example	Your turn
A particle P is moving in a plane so that, at time t seconds, its acceleration is $(3\boldsymbol{i} - 4t\boldsymbol{j})\ ms^{-2}$. When $t = 2$, the velocity of P is $-3\boldsymbol{j}\ ms^{-1}$ and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find: (a) the angle between the direction of motion of P and $\boldsymbol{j}$ when $t = 3$ (b) the distance of P from O when $t = 0$.	A particle P is moving in a plane so that, at time t seconds, its acceleration is $(4\boldsymbol{i} - 2t\boldsymbol{j})\ ms^{-2}$. When $t = 3$, the velocity of P is $6\boldsymbol{i}\ ms^{-1}$ and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find: (a) the angle between the direction of motion of P and $\boldsymbol{i}$ when $t = 2$ (b) the distance of P from O when $t = 0$. a) 68.2° (1 dp) b) 25 m

Worked example	Your turn
The velocity of a particle P at time t seconds is $\left((6t^2-4)\boldsymbol{i}+10\boldsymbol{j}\right)\,ms^{-1}$. When $t=0$, the position vector of P with respect to a fixed origin O is $(5\boldsymbol{i}-3\boldsymbol{j})\,m$. A second particle Q moves with constant velocity $(3\boldsymbol{i}+5\boldsymbol{j})\,ms^{-1}$. When $t=0$, the position vector of Q with respect to the fixed origin O is $2\boldsymbol{j}\,m$. Prove that P and Q collide.	The velocity of a particle P at time t seconds is $\left((3t^2-8)\boldsymbol{i}+5\boldsymbol{j}\right)\,ms^{-1}$. When $t=0$, the position vector of P with respect to a fixed origin O is $(2\boldsymbol{i}-4\boldsymbol{j})\,m$. A second particle Q moves with constant velocity $(8\boldsymbol{i}+4\boldsymbol{j})\,ms^{-1}$. When $t=0$, the position vector of Q with respect to the fixed origin O is $2\boldsymbol{i}\,m$. Prove that P and Q collide. Proof