7.7) Modelling with trigonometric functions

Worked example

The cabin pressure, P (psi) on an aeroplane at cruising altitude can be modelled by the equation

$$P = 14.5 - 0.2\sin(t - 3)$$

where t is the time in hours since cruising altitude was first reached, and angles are in radians. Find:

a) The maximum and minimum cabin pressure
b) The time after reaching cruising altitude that the cabin first reaches a maximum pressure
c) The cabin pressure after 3 hours at cruising altitude
d) All the times within the first 10 hours of cruising that the cabin pressure would be exactly 14.42 psi

Your turn

The cabin pressure, P (psi) on an aeroplane at cruising altitude can be modelled by the equation

$$P = 11.5 - 0.5\sin(t - 2)$$

where t is the time in hours since cruising altitude was first reached, and angles are in radians. Find:

a) The maximum and minimum cabin pressure
b) The time after reaching cruising altitude that the cabin first reaches a maximum pressure
c) The cabin pressure after 5 hours at cruising altitude
d) All the times within the first 8 hours of cruising that the cabin pressure would be exactly 11.3 psi

a) Maximum = 12 psi ; Minimum = 11 psi
b) 0.43 hours = 26 minutes
c) 11.43 psi
d) 2 hours 25 minutes, and 4 hours 44 minutes

Worked example

a) Express $7\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $7\cos\theta - 5\sin\theta$ and the value of θ, for $0 < \theta < 2\pi$ at which this maximum occurs.

The height H above ground of a passenger on a Ferris wheel is modelled by the equation

$$H = 12 - 7\cos(\frac{\pi t}{4}) + 5\sin(\frac{\pi t}{4})$$

where H is measured in metres, and t is the time in minutes after the wheel starts turning.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum first occurs

d) Determine the time for the Ferris wheel to complete five revolutions

Your turn

a) Express $9\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $9\cos\theta - 2\sin\theta$ and the value of θ, for $0 < \theta < 2\pi$ at which this maximum occurs.

The height H above ground of a passenger on a Ferris wheel is modelled by the equation

$$H = 10 - 9\cos(\frac{\pi t}{5}) + 2\sin(\frac{\pi t}{5})$$

where H is measured in metres, and t is the time in minutes after the wheel starts turning.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum first occurs

d) Determine the time for the Ferris wheel to complete two revolutions

a) $R = \sqrt{85}, \alpha = 0.2187$

b) Maximum $= \sqrt{85}$ when $\theta = 6.06$

c) Maximum $H = 19.22m$ at $t = 4.65$

d) 20 minutes

Worked example

a) Express $1.5\sin\theta - 2\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of$1.5\sin\theta - 2\cos\theta$and the value of θ, for $0 < \theta < \pi$ at which this maximum occurs.

The height H of sea water on a particular day can be modelled by the equation

$$H = 8 + 1.5\sin\left(\frac{2\pi t}{25}\right) - 2\cos\left(\frac{2\pi t}{25}\right), 0 \le t < 12$$

where H is measured in metres, and t is the number of hours after midnight.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum occurs

d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

Your turn

a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $2\sin\theta - 1.5\cos\theta$ and the value of θ, for $0 < \theta < \pi$ at which this maximum occurs.

The height H of sea water on a particular day can be modelled by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), 0 \le t < 12$$

where H is measured in metres, and t is the number of hours after midday.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum occurs

d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

a) $R = 2.5, \alpha = 0.6435$

b) Maximum $= 2.5$ when $\theta = 2.21$

c) Maximum $H = 8.5m$ at $t = 4.41$

d) $14{:}06$ and $18{:}43$