

7.5) Simplifying $a \cos x \pm b \sin x$

Worked example

Express $5 \sin x + 12 \cos x$ in the form:
 $R \sin(x + \alpha)$, $R > 0$, $0 < \alpha < 90^\circ$

Your turn

Express $3 \sin x + 4 \cos x$ in the form:
 $R \sin(x + \alpha)$, $R > 0$, $0 < \alpha < 90^\circ$

$$5 \sin(x + 53.1^\circ)$$

Worked example

Express $5 \sin x + 12 \cos x$ in the form:

$$R \cos(x - \alpha), R > 0, 0 < \alpha < 90^\circ$$

Your turn

Express $3 \sin x + 4 \cos x$ in the form:

$$R \cos(x - \alpha), R > 0, 0 < \alpha < 90^\circ$$

$$5 \cos(x - 36.9^\circ)$$

Worked example

Express $5 \sin x + 12 \cos x$ in the form:
 $R \sin(x - \alpha), R > 0, 0 < \alpha < 180^\circ$

Your turn

Express $3 \sin x + 4 \cos x$ in the form:
 $R \sin(x - \alpha), R > 0, 0 < \alpha < 180^\circ$

$$5 \sin(x + 126.9^\circ)$$

Worked example

Express $5 \sin x + 12 \cos x$ in the form:
 $R \cos(x + \alpha)$, $R > 0$, $0 < \alpha < 180^\circ$

Your turn

Express $3 \sin x + 4 \cos x$ in the form:
 $R \cos(x + \alpha)$, $R > 0$, $0 < \alpha < 180^\circ$

$$5 \cos(x + 143.1^\circ)$$

Worked example

Solve in the interval $0 < \theta < 360^\circ$:

$$5 \cos \theta + 2 \sin \theta = 3$$

Your turn

Solve in the interval $0 < \theta < 360^\circ$:

$$2 \cos \theta + 5 \sin \theta = 3$$

$$\theta = 12.1^\circ, 124.3^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq \theta < 180^\circ$:

$$5 \sin 3\theta - 12 \cos 3\theta = 1$$

Your turn

Solve in the interval $0 \leq \theta < 180^\circ$:

$$3 \sin 3\theta - 4 \cos 3\theta = 1$$

$$\theta = 21.6^\circ, 73.9^\circ, 141.6^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq \theta < 180^\circ$:

$$5 \cos 3\theta - 12 \sin 3\theta = 1$$

Your turn

Solve in the interval $0 \leq \theta < 180^\circ$:

$$3 \cos 3\theta - 4 \sin 3\theta = 1$$

$$\theta = 8.4^\circ, 76.1^\circ, 128.4^\circ \text{ (1 dp)}$$

Worked example

Solve in the interval $0 \leq \theta < 360^\circ$:

$$\cot \theta + 4 = \operatorname{cosec} \theta$$

Your turn

Solve in the interval $0 \leq \theta < 360^\circ$:

$$\cot \theta + 3 = \operatorname{cosec} \theta$$

$$\theta = 143.1^\circ \text{ (1 dp)}$$

Worked example

Find the maximum value and the smallest positive value of θ at which the maximum occurs for:

$$4 \cos \theta + 3 \sin \theta$$

Your turn

Find the maximum value and the smallest positive value of θ at which the maximum occurs for:

$$12 \cos \theta + 5 \sin \theta$$

Maximum = 13 when $\theta = 22.6^\circ$ (1 dp)

Worked example

Find the minimum value and the smallest positive value of θ at which the minimum occurs for:

$$4 \cos \theta + 3 \sin \theta$$

Your turn

Find the minimum value and the smallest positive value of θ at which the minimum occurs for:

$$12 \cos \theta + 5 \sin \theta$$

Minimum = -13 when $\theta = 202.6^\circ$ (1 dp)

Worked example

Find the maximum value and the smallest positive value of θ at which the maximum occurs for:

$$\frac{5}{7 + 4 \cos \theta - 3 \sin \theta}$$

Your turn

Find the maximum value and the smallest positive value of θ at which the maximum occurs for:

$$\frac{3}{17 + 12 \cos \theta - 5 \sin \theta}$$

$$\text{Maximum} = \frac{3}{4} \text{ when } \theta = 157.4^\circ \text{ (1 dp)}$$

Worked example

Find the minimum value and the smallest positive value of θ at which the minimum occurs for:

$$\frac{5}{7 + 4 \cos \theta - 3 \sin \theta}$$

Your turn

Find the minimum value and the smallest positive value of θ at which the minimum occurs for:

$$\frac{3}{17 + 12 \cos \theta - 5 \sin \theta}$$

$$\text{Minimum} = \frac{1}{10} \text{ when } \theta = 337.4^\circ \text{ (1 dp)}$$