7.5) Methods of proof

Worked example	Your turn
Prove by exhaustion that the sum of an integer and the square of the integer is even for all integers n	Prove by exhaustion that all square numbers are either a multiple of 4 or 1 more than a multiple of 4 Proof

Worked example	Your turn
Disprove the statement: "$n^2 - n + 3$ is prime for all integers n."	Disprove the statement: "$n^2 - n + 41$ is prime for all integers n." $n = 41, n^2 - n + 41 = 41^2 = 1681$ 1681 has 3 factors: $1, 41$ and 1681

Worked example	Your turn
a) Prove that for all positive values of p and q $$p + q > \sqrt{4pq}$$ b) Use a counter-example to show that this is not true when p and q are not both positive.	a) Prove that for all positive values of x and y $$\frac{x}{y} + \frac{y}{x} \geq 2$$ b) Use a counter-example to show that this is not true when x and y are not both positive.

a) $$(x - y)^2 \geq 0$$
$$x^2 + y^2 - 2xy \geq 0$$
$$\frac{x^2 + y^2 - 2xy}{xy} \geq 0$$
[valid as $x, y > 0 \therefore xy > 0$]
$$\frac{x^2}{xy} + \frac{y^2}{xy} - \frac{2xy}{xy} \geq 0$$
$$\frac{x}{y} + \frac{y}{x} - 2 \geq 0$$
$$\frac{x}{y} + \frac{y}{x} \geq 2$$

b) e.g. $x = -3, y = 6$
$$-\frac{3}{6} + \frac{6}{-3} = -\frac{1}{2} - 2 = -\frac{5}{2} < -2$$