7.4) Two-tailed tests

Worked example	Your turn
Joan believes the probability of rolling a 4 on a six-sided dice is $\frac{1}{6}$. She rolls the dice 10 times and rolls a 4 five times. Using a 5% significance level, test her belief.	John believes a coin is lands on tails with probability $\frac{1}{2}$. He tosses the coin 8 times and it lands on tails 8 times. Using a 5% significance level, test his belief. X = number of times coin lands on tails. p = probability/proportion of times coin lands on tails. $H_0: p = 0.5$ $H_1: p \neq 0.5$ Assume H_0 true. $X \sim B(8, 0.5)$ 5% significance level Reject H_0 if $P(X \geq 8) < 0.05$ Test $P(X \geq 8) = 0.0039 \ldots < 0.05$ The result is significant. Sufficient evidence to reject H_0 Sufficient evidence to reject John's belief

Worked example	Your turn
Joan believes the probability of rolling a 4 on a six-sided dice is $\frac{1}{6}$. She rolls the dice 10 times. a) Find the critical region(s) for this test at the 5% significance level. b) A 4 is rolled five times. Comment on this observation in light of the critical region.	John believes a coin is lands on tails with probability $\frac{1}{2}$. He tosses the coin 8 times. a) Find the critical region(s) for this test at the 5% significance level. b) The coin lands on tails 8 times. Comment on this observation in light of the critical region.

a) X = number of times coin lands on tails.
p = probability/proportion of times coin lands on tails.
$H_0: p = 0.5$
$H_1: p \neq 0.5$
Assume H_0 true. $X \sim B(8, 0.5)$
5% significance level
Reject H_0 if $P(X \leq x_1) < 0.025$ or $P(X \geq x_2) < 0.025$

$P(X \leq 1) = 0.0351 \ldots > 0.025$
$P(X \leq 0) = 0.0039 \ldots < 0.025$
$\therefore x_1 = 0$

$1 - P(X \leq x_2 - 1) < 0.025$
$-P(X \leq x_2 - 1) < -0.975$
$P(X \leq x_2 - 1) > 0.975$
$P(X \leq 6) = 0.9648 \ldots < 0.975$
$P(X \leq 7) = 0.9960 \ldots > 0.975$
$x_2 - 1 = 7$
$\therefore x_2 = 8$

Lower tail: $X = 0$
Upper tail $X = 8$

Critical regions: Reject H_0 if $X = 0 \cup X = 8$

b) 8 is in the critical region.
The result is significant.
Sufficient evidence to reject H_0
Sufficient evidence to reject John's belief

Worked example	Your turn
An election candidate believes he has the support of 30% of the residents in a particular town. The researcher asks 30 people whether they support the candidate or not. 1 person says they support the candidate. Test, at the 1% significance level, whether the candidate's claim is true.	An election candidate believes she has the support of 40% of the residents in a particular town. The researcher asks 20 people whether they support the candidate or not. 3 people say they support the candidate. Test, at the 2% significance level, whether the candidate's claim is true. X = number of people who say they support the candidate p = probability/proportion of people who say they support the candidate $H_0: p = 0.4$ $H_1: p \neq 0.4$ Assume H_0 true. $X \sim B(20, 0.4)$ 5% significance level Reject H_0 if $P(X \leq 3) < 0.01$ Test $P(X \leq 3) = 0.0159 \ldots > 0.01$ The result is not significant. Insufficient evidence to reject H_0 Insufficient evidence to reject the candidate's belief.

# Worked example	# Your turn
An election candidate believes he has the support of 30% of the residents in a particular town. The researcher asks 30 people whether they support the candidate or not. a) Find the critical region(s) for a test of the candidate's claim at the 1% significance level. b) 1 person says they support the candidate. Comment on this observation in light of the critical region.	An election candidate believes she has the support of 40% of the residents in a particular town. The researcher asks 20 people whether they support the candidate or not. a) Find the critical region(s) for a test of the candidate's claim at the 2% significance level. b) 3 people say they support the candidate. Comment on this observation in light of the critical region.

X = number of people who say they support the candidate
p = probability/proportion of people who say they support the candidate
$H_0: p = 0.4$
$H_1: p \neq 0.4$
Assume H_0 true. $X \sim B(20, 0.4)$
5% significance level

Reject H_0 if $P(X \leq x_1) < 0.01$ or $P(X \geq x_2) < 0.01$

$P(X \leq 3) = 0.0159 \ldots > 0.01$
$P(X \leq 2) = 0.0036 \ldots < 0.01$
$\therefore x_1 = 2$

$1 - P(X \leq x_2 - 1) < 0.01$
$-P(X \leq x_2 - 1) < -0.99$
$P(X \leq x_2 - 1) > 0.99$
$P(X \leq 12) = 0.9789 \ldots < 0.99$
$P(X \leq 13) = 0.9935 \ldots > 0.99$
$x_2 - 1 = 13$
$\therefore x_2 = 14$

Lower tail: $0 \leq X \leq 2$ Upper tail $14 \leq X \leq 20$

Critical regions: Reject H_0 if $0 \leq X \leq 2 \cup 14 \leq X \leq 20$

b) 3 is not in the critical region.
The result is not significant.
Insufficient evidence to reject H_0
Insufficient evidence to reject the candidate's belief.

Worked example	Your turn
An election candidate believes he has the support of 30% of the residents in a particular town. The researcher asks 30 people whether they support the candidate or not. 16 people say they support the candidate. Test, at the 1% significance level, whether the candidate's claim is true.	An election candidate believes she has the support of 40% of the residents in a particular town. The researcher asks 20 people whether they support the candidate or not. 14 people say they support the candidate. Test, at the 2% significance level, whether the candidate's claim is true. X = number of people who say they support the candidate p = probability/proportion of people who say they support the candidate $H_0: p = 0.4$ $H_1: p \neq 0.4$ Assume H_0 true. $X \sim B(20, 0.4)$ 5% significance level Reject H_0 if $P(X \geq 14) < 0.01$ Test $P(X \geq 14) = 0.0064 \ldots < 0.01$ The result is significant. Sufficient evidence to reject H_0 Sufficient evidence to reject the candidate's belief.

Worked example	Your turn
An election candidate believes he has the support of 30% of the residents in a particular town. The researcher asks 30 people whether they support the candidate or not. a) Find the critical region(s) for a test of the candidate's claim at the 1% significance level. b) 16 people say they support the candidate. Comment on this observation in light of the critical region.	An election candidate believes she has the support of 40% of the residents in a particular town. The researcher asks 20 people whether they support the candidate or not. a) Find the critical region(s) for a test of the candidate's claim at the 2% significance level. b) 14 people say they support the candidate. Comment on this observation in light of the critical region.

$X =$ number of people who say they support the candidate
$p =$ probability/proportion of people who say they support the candidate
$H_0: p = 0.4$
$H_1: p \neq 0.4$
Assume H_0 true. $X \sim B(20, 0.4)$
5% significance level

Reject H_0 if $P(X \leq x_1) < 0.01$
$P(X \leq 3) = 0.0159 \ldots > 0.01$
$P(X \leq 2) = 0.0036 \ldots < 0.01$
$\therefore x_1 = 2$

or $P(X \geq x_2) < 0.01$
$1 - P(X \leq x_2 - 1) < 0.01$
$-P(X \leq x_2 - 1) < -0.99$
$P(X \leq x_2 - 1) > 0.99$
$P(X \leq 12) = 0.9789 \ldots < 0.99$
$P(X \leq 13) = 0.9935 \ldots > 0.99$
$x_2 - 1 = 13$
$\therefore x_2 = 14$

Lower tail: $0 \leq X \leq 2$
Upper tail $14 \leq X \leq 20$

Critical regions: Reject H_0 if $0 \leq X \leq 2 \cup 14 \leq X \leq 20$

b) 14 is not in the critical region.
The result is significant.
Sufficient evidence to reject H_0.
Sufficient evidence to reject the candidate's belief.

Worked example	Your turn
In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08. The manufacturing process is changed. A sample of 200 lightbulbs is tested. 7 lightbulbs are found to be faulty. Test, at the 2% significance level, whether or not there has been a change in the proportion of faulty lightbulbs.	In a manufacturing process, the proportion of faulty bolts is, based on historical data, 0.07. The manufacturing process is changed. A sample of 100 bolts is tested. 1 bolt is found to be faulty. Test, at the 1% significance level, whether or not there has been a change in the proportion of faulty bolts. X = number of fault bolts p = probability/proportion of faulty bolts $H_0: p = 0.07$ $H_1: p < 0.07$ Assume H_0 true. $X \sim B(100, 0.07)$ 1% significance level Reject H_0 if $P(X \leq 1) < 0.005$ Test $P(X \leq 1) = 0.0060 \ldots > 0.005$ The result is not significant. Insufficient evidence to reject H_0 Insufficient evidence to suggest there has been a change in the proportion of faulty bolts.

Worked example	Your turn
In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08. The manufacturing process is changed. The manager wants to test whether or not the proportion of faulty lightbulbs has changed. A sample of 200 lightbulbs is tested. a) Find the critical region(s) for a test at the 2% significance level. b) 7 lightbulbs are found to be faulty. Comment on this observation in light of the critical region.	In a manufacturing process, the proportion of faulty bolts is, based on historical data, 0.07. The manufacturing process is changed. The manager wants to test whether or not the proportion of faulty bolts has changed. A sample of 100 bolts is tested. a) Find the critical region(s) for a test at the 1% significance level. b) 1 bolt is found to be faulty. Comment on this observation in light of the critical region.

a) X = number of fault bolts

p = probability/proportion of faulty bolts

$H_0: p = 0.07$

$H_1: p < 0.07$

Assume H_0 true. $X \sim B(100, 0.07)$

1% significance level

Reject H_0 if $P(X \leq x_1) < 0.005$ or $P(X \geq x_2) < 0.005$

$P(X \leq 1) = 0.0060 \ldots > 0.005$

$P(X \leq 0) = 0.0007 \ldots < 0.005$

$\therefore x_1 = 0$

$1 - P(X \leq x_2 - 1) < 0.005$

$-P(X \leq x_2 - 1) < -0.995$

$P(X \leq x_2 - 1) > 0.995$

$P(X \leq 13) = 0.9900 \ldots < 0.995$

$P(X \leq 14) = 0.9959 \ldots > 0.995$

$x_2 - 1 = 14$

$\therefore x_2 = 15$

Lower tail: $X = 0$

Upper tail $15 \leq X \leq 100$

Critical regions: Reject H_0 if $X = 0 \cup 15 \leq X \leq 100$

b) 1 is not in the critical region.

The result is not significant.

Insufficient evidence to reject H_0

Insufficient evidence to suggest the proportion of fault bolts has changed.

Worked example

A medical team are testing the effectiveness of a new drug.
They claim that the test is successful 99.8% of the time.
They test the benefits of the drug on 4500 patients.
The test is successful in 4498 cases.
Is the medical team's claim supported at the 1% significance level?

Your turn

A medical team are testing the effectiveness of a new drug.
They claim that the test is successful 99.5% of the time.
They test the benefits of the drug on 2500 patients.
The test is successful in 2495 cases.
Is the medical team's claim supported at the 5% significance level?

$X =$ number of successful tests
$p =$ probability/proportion of successful tests
$H_0: p = 0.995$
$H_1: p \neq 0.995$
Under H_0, $X \sim B(2500, 0.995)$
5% significance level
Reject H_0 if $P(X \geq 2495) < 0.025$
Test $P(X \geq 2495) = 0.01464 \ldots < 0.025$
The result is significant.
Sufficient evidence to reject H_0
Sufficient evidence to reject the medical team's claim.

Worked example

A medical team are testing the effectiveness of a new drug.
They claim that the test is successful 99.8% of the time.
They test the benefits of the drug on 4500 patients.

a) Find the critical region(s) for a test at the 1% significance level.
b) The test is successful in 4498 cases. Comment on this observation in light of the critical region.

Your turn

A medical team are testing the effectiveness of a new drug.
They claim that the test is successful 99.5% of the time.
They test the benefits of the drug on 2500 patients.

a) Find the critical region(s) for a test at the 5% significance level.
b) The test is successful in 2495 cases. Comment on this observation in light of the critical region.

a) X = number of successful tests
p = probability/proportion of successful tests
$H_0: p = 0.995$
$H_1: p \neq 0.995$
Under H_0, $X \sim B(2500, 0.995)$
5% significance level
Reject H_0 if $P(X \leq x_1) < 0.025$ or $P(X \geq x_2) < 0.025$

$P(X \leq 2480) = 0.0302 \ldots > 0.025$
$P(X \leq 2479) = 0.0170 \ldots < 0.025$
$\therefore x_1 = 2479$

$1 - P(X \leq x_2 - 1) < 0.025$
$-P(X \leq x_2 - 1) < -0.975$
$P(X \leq x_2 - 1) > 0.975$
$P(X \leq 2493) = 0.9657 \ldots < 0.975$
$P(X \leq 2494) = 0.9853 \ldots > 0.975$
$x_2 - 1 = 2494$
$\therefore x_2 = 2495$

Lower tail: $0 \leq X \leq 2479$ Upper tail $2495 \leq x \leq 2500$

Critical regions: Reject H_0 if $0 \leq X \leq 2479 \cup 2495 \leq x \leq 2500$

b) 2495 is in the critical region.
The result is significant.
Sufficient evidence to reject H_0
Sufficient evidence to reject the medical team's claim.

Worked example	Your turn
A medical team are testing the effectiveness of a new drug. They claim that the test is successful 99.8% of the time. They test the benefits of the drug on 4500 patients. a) Find the critical region(s) for a test at the 1% significance level. b) The test is successful in 4482 cases. Comment on this observation in light of the critical region.	A medical team are testing the effectiveness of a new drug. They claim that the test is successful 99.5% of the time. They test the benefits of the drug on 2500 patients. a) Find the critical region(s) for a test at the 5% significance level. b) The test is successful in 2480 cases. Comment on this observation in light of the critical region.

a) X = number of successful tests

p = probability/proportion of successful tests

$H_0: p = 0.995$

$H_1: p \neq 0.995$

Under H_0, $X \sim B(2500, 0.995)$

5% significance level

Reject H_0 if $P(X \leq x_1) < 0.025$ or $P(X \geq x_2) < 0.025$

$P(X \leq 2480) = 0.0302 \ldots > 0.025$

$P(X \leq 2479) = 0.0170 \ldots < 0.025$

$\therefore x_1 = 2479$

$1 - P(X \leq x_2 - 1) < 0.025$

$-P(X \leq x_2 - 1) < -0.975$

$P(X \leq x_2 - 1) > 0.975$

$P(X \leq 2493) = 0.9657 \ldots < 0.975$

$P(X \leq 2494) = 0.9853 \ldots > 0.975$

$x_2 - 1 = 2494$

$\therefore x_2 = 2495$

Lower tail: $0 \leq X \leq 2479$

Upper tail $2495 \leq x \leq 2500$

Critical regions: Reject H_0 if $0 \leq X \leq 2479 \cup 2495 \leq x \leq 2500$

b) 2480 is not in the critical region.

The result is not significant.

Insufficient evidence to reject H_0

Insufficient evidence to reject the medical team's claim.