7C Particular Integrals of Second Order Differential Equations

$$\boldsymbol{a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)}$$

1. Find the solution of the differential equation:

a)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3$$

b)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2x$$

c)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3x^2$$

d)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x$$

e)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 13sin3x$$

2. Find the general solution to the following differential equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3$$

Summary:

1) Start by finding the Complimentary Function by setting the differential equation equal to 0, then forming the auxiliary equation (as in the previous sections)

2) Find the Particular Integral by considering f(x) and letting y equal something of the same form. Then differentiate it and replace these in the original equation and solve for the unknowns

→ Use the table to the right (which you are NOT given…)

3) Combine the CF and PI to create the equation in y

Form of f(x)	Form of PI
k	λ
kx	λx + μ
kx^2	$\lambda x^2 + \mu x + \nu$
ke^{px}	λe^{px}
mcosax	λcosax + μsinax
msinax	λcosax + μsinax
mcosax+nsinax	λcosax + μsinax

If the form of the Particular Integral is already in the Complimentary Function, include an 'x' in it as well (as we did on the last example!)