Second Order Non-Homogenous DE's

Non – homogeneous, second order DE's have the form

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=\boldsymbol{f(x)}$$

Question: Start by considering the solution to the 1st order DE: $\frac{dy}{dx}+2y=\mathbf{3}\boldsymbol{x}-\mathbf{1}$

Consider another example with the same LHS: $\frac{dy}{dx} + 2y = \boldsymbol{e^{3x}}$

Solving a Second Order, Non – Homogeneous DE

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = \boldsymbol{f(x)}$$

1. Solve $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = \mathbf{0}$ first to obtain what is known as the **complementary function**. (C.F.)

2. Then solve $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = \boldsymbol{f(x)}$ which can be found using appropriate substitution and comparing coefficients. Solution known as particular integral. (P.I.)

3.

$$\boldsymbol{y = C.F. + P.I.}$$

This is because $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy$ for the C.F. is 0 and $f(x)$ for the P.I., which sum to $f(x)$

How do we find the Particular Integral?

To find a particular integral you need to establish a **trial function** whose form depends on the form of $f(x)$.

Function ($f(x)$)	Form of Particular Integral
p	λ
$p + qx$	$\lambda + \mu x$
$p + qx + rx^2$	$\lambda + \mu x + vx^2$
pe^{kx}	λe^{kx}
$pcos\omega x + qsin\omega x$	$\lambda cos\omega x + \mu sin\omega x$
$pcos\omega x$	$\lambda cos\omega x + \mu sin\omega x$

The Particular Integral is a function which satisfies the original DE. We take our trial form and sub it back into the DE to find the value of the coefficients.

Example

Find the **particular integral** of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \mathbf{3}$

Example:

Find the **general solution** of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \mathbf{2x}$

Example:

Find the **general solution** of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \mathbf{3x^2}$

Example:

Find the **general solution** of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \boldsymbol{e^x}$

<u>Interesting (and important) Example!</u>

Find the **general solution** of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \boldsymbol{e^{2x}}$

Example:

Find the **general solution** of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = \mathbf{3}$