

## Second Order Non-Homogenous DE's

Non – homogeneous, second order DE's have the form

$$a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$$

Question: Start by considering the solution to the 1<sup>st</sup> order DE:

$$\frac{dy}{dx} + 2y = 3x - 1$$

Consider another example with the same LHS:  $\frac{dy}{dx} + 2y = e^{3x}$



## Solving a Second Order, Non – Homogeneous DE

$$a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$$

1. Solve  $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$   
first to obtain what is known as  
the **complementary function**.  
(C.F.)

2. Then solve  $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$   
which can be found using appropriate  
substitution and comparing  
coefficients. Solution known as  
particular integral. (P.I.)

3.

$$y = C.F. + P.I.$$

This is because  $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy$   
for the C.F. is 0 and  $f(x)$  for the P.I.,  
which sum to  $f(x)$

### How do we find the Particular Integral?

To find a particular integral you need to establish a **trial function** whose form depends on the form of  $f(x)$ .

Function ( $f(x)$ )	Form of Particular Integral
$p$	$\lambda$
$p + qx$	$\lambda + \mu x$
$p + qx + rx^2$	$\lambda + \mu x + \nu x^2$
$pe^{kx}$	$\lambda e^{kx}$
$p \cos \omega x + q \sin \omega x$	$\lambda \cos \omega x + \mu \sin \omega x$
$p \cos \omega x$	$\lambda \cos \omega x + \mu \sin \omega x$

The Particular Integral is a function which satisfies the original DE. We take our trial form and sub it back into the DE to find the value of the coefficients.

Example

Find the **particular integral** of the differential equation  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3$

Example:

Find the **general solution** of the differential equation  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2x$

Example:

Find the **general solution** of the differential equation  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3x^2$

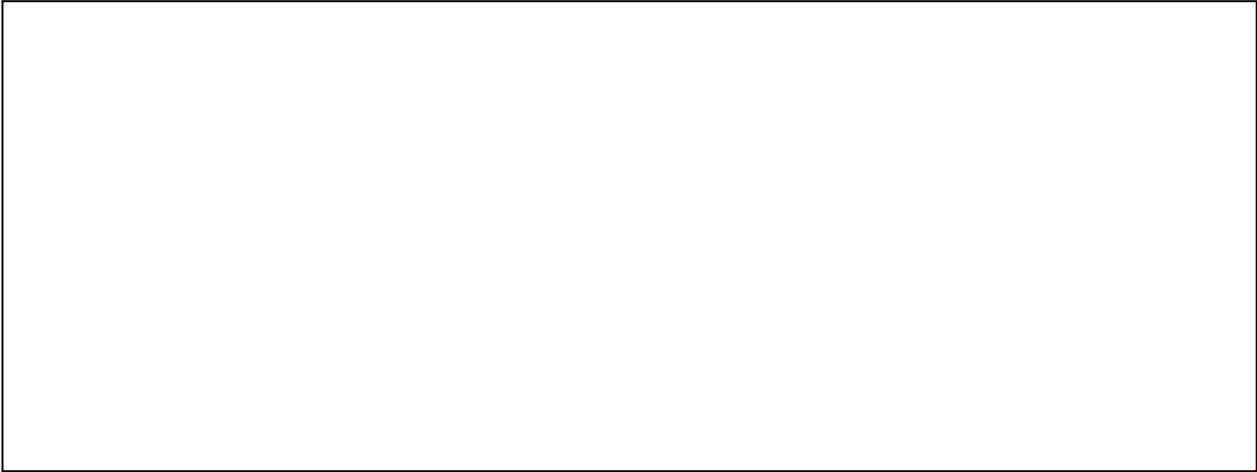
Example:

Find the **general solution** of the differential equation  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x$

**Interesting (and important) Example!**

Find the **general solution** of the differential equation  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$





Example:

Find the **general solution** of the differential equation  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3$