Second Order Non-Homogenous DE’s

Non – homogeneous, second order DE’s have the form

$$a\frac{d^{2}y}{dx^{2}}+b\frac{dy}{dx}+cy=f(x)$$

Question: Start by considering the solution to the 1st order DE: $\frac{ dy}{dx}+2y=3x-1$

Consider another example with the same LHS: $\frac{ dy}{dx}+2y=e^{3x}$

Solving a Second Order, Non – Homogeneous DE



How do we find the Particular Integral?

To find a particular integral you need to establish a **trial function** whose form depends on the form of $f\left(x\right).$

|  |  |
| --- | --- |
| Function ($f\left(x\right))$ | Form of Particular Integral |
| $$p$$ | $$λ$$ |
| $$p+qx$$ | $$λ+µx$$ |
| $$p+qx+rx^{2}$$ | $$λ+µx+vx^{2}$$ |
| $$pe^{kx}$$ | $$λe^{kx}$$ |
| $$pcosωx+qsinωx$$ | $$λcosωx+µsinωx$$ |
| $$pcosωx$$ | $$λcosωx+µsinωx$$ |

The Particular Integral is a function which satisfies the original DE. We take our trial form and sub it back into the DE to find the value of the coefficients.

Example

Find the **particular integral** of the differential equation $\frac{d^{2}y}{dx^{2}}-5\frac{dy}{dx}+6y=3$

Example:

Find the **general solution** of the differential equation $\frac{d^{2}y}{dx^{2}}-5\frac{dy}{dx}+6y=2x$

Example:

Find the **general solution** of the differential equation $\frac{d^{2}y}{dx^{2}}-5\frac{dy}{dx}+6y=3x^{2}$

Example:

Find the **general solution** of the differential equation $\frac{d^{2}y}{dx^{2}}-5\frac{dy}{dx}+6y=e^{x}$

**Interesting (and important) Example!**

Find the **general solution** of the differential equation $\frac{d^{2}y}{dx^{2}}-5\frac{dy}{dx}+6y=e^{2x}$

Example:

Find the **general solution** of the differential equation $\frac{d^{2}y}{dx^{2}}-2\frac{dy}{dx}=3$