7.3) Double-angle formulae

Worked example

Using the trigonometric angle addition formulae, derive:

$$\sin 2x$$

$$\tan 2x$$

Your turn

Using the trigonometric angle addition formulae find:

$$\cos 2x$$

$$\cos^2 x - \sin^2 x$$

Worked example	Your turn
Using $cos2x \equiv \cos^2 x - \sin^2 x$, express $\cos 2x$ using only terms of $\cos^2 x$ and constants	Using $cos2x \equiv \cos^2 x - \sin^2 x$, express $\cos 2x$ using only terms of $\sin^2 x$ and constants $1 - 2\sin^2 x$

Worked example

Use the double-angle formulae to write as a single trigonometric ratio:

$$\cos^2 50° - \sin^2 50°$$

$$2\cos^2 \frac{2\pi}{9} - 1$$

$$1 - 2\sin^2 30°$$

Your turn

Use the double-angle formulae to write as a single trigonometric ratio:

$$\cos^2 15° - \sin^2 15°$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$2\cos^2 22.5° - 1$$

$$\cos 45° = \frac{\sqrt{2}}{2}$$

$$1 - 2\sin^2 \frac{\pi}{4}$$

$$\cos \frac{\pi}{2} = 0$$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio: $\cos^2 2x - \sin^2 2x$	Use the double-angle formulae to write as a single trigonometric ratio: $\cos^2 5x - \sin^2 5x$ $\cos 10x$
$4\cos^2 3x - 2$	$8\cos^2 6x - 4$ $4\cos 12x$
$3 - 6\sin^2 4x$	$5 - 10\sin^2 7x$ $5\cos 14x$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio:	Use the double-angle formulae to write as a single trigonometric ratio:
$2 \sin 45° \cos 45°$	$2 \sin 30° \cos 30°$
	$\sin 60° = \frac{\sqrt{3}}{2}$
$4 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$	$8 \sin \frac{\pi}{4} \cos \frac{\pi}{4}$
	$16 \sin \frac{\pi}{2} = 16$
$7 \sin 5x \cos 5x$	$5 \sin 7x \cos 7x$
	$10 \sin 14x$

Worked example

Use the double-angle formulae to write as a single trigonometric ratio:

$$\frac{2\tan 30°}{1-\tan^2 30°}$$

$$\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$$

$$\frac{4\tan 6x}{1-\tan^2 6x}$$

Your turn

Use the double-angle formulae to write as a single trigonometric ratio:

$$\frac{2\tan 22.5°}{1-\tan^2 22.5°}$$

$\tan 45° = 1$

$$\frac{2\tan\frac{\pi}{4}}{1-\tan^2\frac{\pi}{4}}$$

0

$$\frac{6\tan 4x}{1-\tan^2 4x}$$

$3\tan 8x$

Worked example

Use the double-angle formulae to write as a single trigonometric ratio:

$$\frac{8 \sin 22.5°}{\sec 22.5°}$$

$$\frac{6 \cos \frac{\pi}{4}}{\operatorname{cosec} \frac{\pi}{4}}$$

Your turn

Use the double-angle formulae to write as a single trigonometric ratio:

$$\frac{4 \sin 30°}{\sec 30°}$$

$$2 \sin 60° = \sqrt{3}$$

Worked example	Your turn
Given that $x = 2\sin\theta$ and $y = 4 - 3cos2\theta$, eliminate θ and express y in terms of x.	Given that $x = 3\sin\theta$ and $y = 3 - 4cos2\theta$, eliminate θ and express y in terms of x. $y = \frac{8x^2}{9} - 1$
Given that $x = 5\cos\theta$ and $y = 6 - 7cos2\theta$, eliminate θ and express y in terms of x.	

Worked example

Given that $\cos x = \frac{5}{8}$ and x is acute, find the exact value of

(a) $\sin 2x$ (b) $\tan 2x$

Your turn

Given that $\cos x = \frac{3}{4}$ and x is acute, find the exact value of

(a) $\sin 2x$ (b) $\tan 2x$

(a) $\frac{3\sqrt{7}}{8}$

(b) $3\sqrt{7}$

Worked example

Using the double-angle formulae, evaluate:

$$\left(\sin\frac{\pi}{3}+\cos\frac{\pi}{3}\right)^2$$

$$\left(\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\right)^2$$

Your turn

Using the double-angle formulae, evaluate:

$$\left(\sin\frac{\pi}{6}+\cos\frac{\pi}{6}\right)^2$$

$$\frac{2+\sqrt{3}}{2}$$

Worked example

Given that $0 < \theta < \pi$, find the value of $\tan\frac{\theta}{2}$ when $\tan\theta = -\frac{3}{4}$

Your turn

Given that $\pi < \theta < \frac{3\pi}{2}$, find the value of $\tan\frac{\theta}{2}$ when $\tan\theta = \frac{3}{4}$

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