## 7B Homogeneous Second Order Differential Equations

$$
a \frac{d^{2} y}{d x^{2}}+b \frac{d y}{d x}+c y=0
$$

Solve the equation:

$$
a \frac{d y}{d x}+b y=0
$$

Case 1: $b^{2}>4 a c$
Find $\frac{d y}{d x}$ and $\frac{d^{2} y}{d x^{2}}$ when

$$
y=A e^{m x}
$$

Find the auxiliary equation for

$$
a \frac{d^{2} y}{d x^{2}}+b \frac{d y}{d x}+c y=0
$$

Summary: when $\mathrm{b}^{2}>4 \mathrm{ac}$ then the solution will be in the form...

1. Find the general solution of the equation:

$$
\frac{d^{2} y}{d x^{2}}-7 \frac{d y}{d x}+12 y=0
$$

## Case 2: $\mathbf{b}^{\mathbf{2}=4 a c}$

Show that

$$
y=(A+B x) e^{3 x}
$$

Satisfies the equation:

$$
\frac{d^{2} y}{d x^{2}}-6 \frac{d y}{d x}+9 y=0
$$

Summary: If $\mathrm{b}^{2}=4 \mathrm{ac}$ then the solution will be in the form...
2. Find the general solution of the equation:

$$
\frac{d^{2} y}{d x^{2}}+8 \frac{d y}{d x}+16 y=0
$$

## Case 3: b $^{\mathbf{2}}<\mathbf{4 a c}$

Find the general solution of the differential equation:

$$
\frac{d^{2} y}{d x^{2}}+16 y=0
$$

3. Find the general solution of the differential equation:

$$
\frac{d^{2} y}{d x^{2}}-6 \frac{d y}{d x}+34 y=0
$$

Summary: If $b^{2}=4 a c$ then the solution will be in the form...

A summary of what you have seen up to this point:
$a \frac{d^{2} y}{d x^{2}}+b \frac{d y}{d x}+c y=0$

$A e^{m_{1} x}+B e^{m_{2} x}$

$$
(A+B x) e^{m x}
$$

$e^{p x}(A \cos q x+B \operatorname{sinq} x)$

If the root is imaginary only, $p=0$ hence $e^{p x}=1$ and will not be included in the answer!

