Second Order Differential Equations (Homogenous)

Higher order DE's are often used in Mechanics to model situations which involve acceleration. Second order differential equations involve the second derivative.

Classification of DE's

Recap of First Order, linear, homogeneous DE's

Consider the general DE: $a\frac{dy}{dx} + by = 0$

Solution:

We could use this general result to “guess” at the solution of a DE:

If $5\frac{dy}{dx} + y = 0$, we can assume that $y = Ae^{\alpha x}$. $\quad Hence\ \frac{dy}{dx} =$

Substituting these expressions back into the original DE gives:

For second order DE's: We will consider 4 different situations for second order DE's in relation to their Auxilary Equation:

1. Two Distinct Real Roots

2. Complex Roots which
    a. purely imaginary
    b. general

3. Repeated Roots

1. Two Distinct Real Roots to Auxilary Equation

Let's 'guess' that the solution of $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ is similar, and of the form Ae^{mx}

Let $y = Ae^{mx}$

- The equation $am^2 + bm + c = 0$ is called the auxiliary equation, and if m is a root of the auxiliary equation then $y = Ae^{mx}$ is a solution of the differential equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

- When the auxiliary equation has **two real distinct roots** α and β, the general solution of the differential equation is $y = Ae^{\alpha x} + Be^{\beta x}$, where A and B are arbitrary constants. The solution involves exponential growth or decay. Initial conditions allow us to find the values of A and B.

Example

Find the general solution of the equation $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 3y = 0$

Test Your Understanding

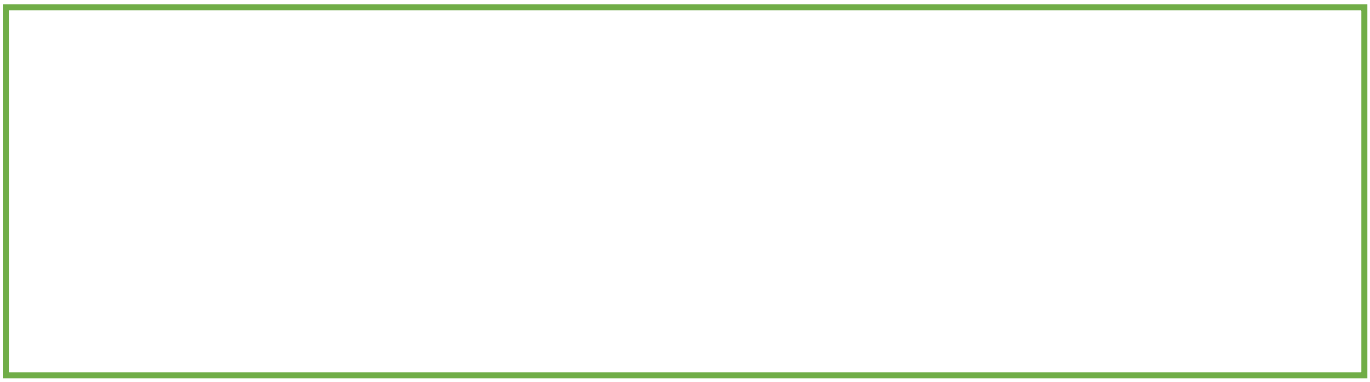
Find the solution of the equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$. Given that when $x = 0,\ \ y = 0,$ $\frac{dy}{dx} = 1.$

2. Two Complex Roots to Auxiliary Equation which are:

   a) Purely imaginary

Example

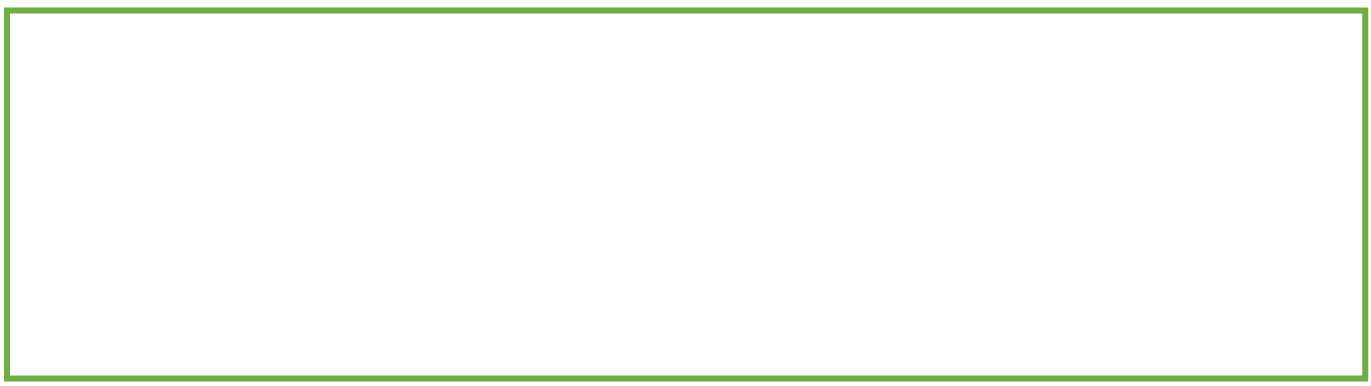
Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 16y = 0$



b. More General Complex Roots

Example

Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 34y = 0$



3. Repeated Roots to Auxiliary Equation

Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Quickfire Questions:

Auxiliary Equation	Roots	General Solution
$m^2 + 6m + 8 = 0$	$m = -2,\ -4$	
$m^2 - 1 = 0$	$m = \pm 1$	
$m^2 - 2m + 1 = 0$	$m = 1$	
$m^2 + 4 = 0$	$m = \pm 2i$	
$m^2 + 10x + 25 = 0$	$m = -5$	
$m^2 - 12m + 45 = 0$	$m = 6 \pm 3i$	
$m^2 + 10 = 0$	$m = \pm\sqrt{10}$	
$m^2 + 2m + 5$	$m = -1 \pm 2i$	