

## 7.2) Finding critical values

## Worked example

Joan believes a six-sided dice is biased in favour of rolling a 4.

She rolls the dice 10 times and counts the number of times,  $X$ , it rolls a 4.

- a) Using a 5% significance level, find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

John believes a coin is biased in favour of landing with tails uppermost.

He tosses the coin 8 times and counts the number of times,  $X$ , it lands with tails uppermost.

- a) Using a 5% significance level, find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $7 \leq X \leq 8$

b) 0.0352 (4 dp)

## Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.

A researcher wants to test, at the 10% significance level, whether the candidate is over-estimating his support.

The researcher asks 30 people whether they support the candidate or not.

- a) Find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.

A researcher wants to test, at the 5% significance level, whether the candidate is over-estimating her support.

The researcher asks 20 people whether they support the candidate or not.

- a) Find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $0 \leq X \leq 3$

b) 0.0160 (4 dp)

## Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.

A researcher wants to test, at the 1% significance level, whether the candidate is under-estimating his support.

The researcher asks 30 people whether they support the candidate or not.

- a) Find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.

A researcher wants to test, at the 2% significance level, whether the candidate is under-estimating her support.

The researcher asks 20 people whether they support the candidate or not.

- a) Find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $14 \leq X \leq 20$

b) 0.0065 (4 dp)

## Worked example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08. A sample of 200 lightbulbs is tested. The manager wishes to test at the 2% significance level whether or not there has been a reduction in the proportion of faulty lightbulbs.

- a) Find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

In a manufacturing process, the proportion of faulty bolts is, based on historical data, 0.07. A sample of 100 bolts is tested. The manager wishes to test at the 1% significance level whether or not there has been a reduction in the proportion of faulty bolts.

- a) Find the critical region for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $0 \leq X \leq 1$

b) 0.0060 (4 dp)

## Worked example

Joan believes the probability of rolling a 4 on a six-sided dice is  $\frac{1}{6}$ .

She rolls the dice 10 times and counts the number of times,  $X$ , it rolls a 4.

Using a 5% significance level,

- find the critical region(s) for this test.
- find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

John believes a coin is lands on tails with probability  $\frac{1}{2}$ .

He tosses the coin 8 times and counts the number of times,  $X$ , it lands with tails uppermost.

Using a 5% significance level,

- find the critical region(s) for this test.
- find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $X = 0 \cup X = 8$

b) 0.0078 (4 dp)

## Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.

A researcher wants to test, at the 10% significance level, whether this claim is true.

The researcher asks 30 people whether they support the candidate or not.

- a) Find the critical region(s) for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.

A researcher wants to test, at the 5% significance level, whether this claim is true.

The researcher asks 20 people whether they support the candidate or not.

- a) Find the critical region(s) for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $0 \leq X \leq 3 \cup 13 \leq X \leq 20$

b) 0.0370 (4 dp)

## Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.

A researcher wants to test, at the 1% significance level, whether this claim is true.

The researcher asks 30 people whether they support the candidate or not.

- a) Find the critical region(s) for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.

A researcher wants to test, at the 2% significance level, whether this claim is true.

The researcher asks 20 people whether they support the candidate or not.

- a) Find the critical region(s) for this test.
- b) Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $0 \leq X \leq 2 \cup 14 \leq X \leq 20$

b) 0.0101 (4 dp)



## Worked example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08. The manufacturing process is changed. A sample of 200 lightbulbs is tested. The manager wishes to test at the 2% significance level whether or not there has been a change in the proportion of faulty lightbulbs.

- Find the critical region(s) for this test.
- Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

## Your turn

In a manufacturing process, the proportion of faulty bolts is, based on historical data, 0.07. The manufacturing process is changed. A sample of 100 bolts is tested. The manager wishes to test at the 1% significance level whether or not there has been a change in the proportion of faulty bolts.

- Find the critical region(s) for this test.
- Find the probability of incorrectly rejecting the null hypothesis (the actual significance level)

a)  $X = 0 \cup 15 \leq X \leq 100$

b) 0.0048 (4 dp)

## Worked example

A random variable has distribution  $B(40, p)$ .  
A single observation is used to test  $H_0: p = 0.1$

against  $H_1: p \neq 0.1$ .

Using a 1% level of significance, find the critical region for this test. The probability in each tail should be as close as possible to 0.005

## Your turn

A random variable has distribution  $B(30, p)$ .  
A single observation is used to test  $H_0: p = 0.2$

against  $H_1: p \neq 0.2$ .

Using a 5% level of significance, find the critical region for this test. The probability in each tail should be as close as possible to 0.025

$$0 \leq X \leq 1 \cup 11 \leq X \leq 30$$