CP2 Chapter 7

Differential Equations

Course Structure

1. First Order Differential Equations (Booklet 1)
2. Second Order Differential Equations – Homogeneous (Booklet 2)
3. Second Order Differentials – Non-homogeneous (Booklet 3)

9 **Differential equations**	9.1	Find and use an integrating factor to solve differential equations of form $\frac{dy}{dx} + P(x)y = Q(x)$ and recognise when it is appropriate to do so.	The integrating factor $e^{\int P(x)dx}$ may be quoted without proof.
	9.2	Find both general and particular solutions to differential equations.	Students will be expected to sketch members of the family of solution curves.
	9.3	Use differential equations in modelling in kinematics and in other contexts.	
	9.4	Solve differential equations of form $y'' + ay' + by = 0$ where a and b are constants by using the auxiliary equation.	

9 **Differential equations** ***continued***	9.5	Solve differential equations of form $y''+a\,y'+b\,y=\mathrm{f}(x)$ where a and b are constants by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $\mathrm{f}(x)$ is a polynomial, exponential or trigonometric function).	$\mathrm{f}(x)$ will have one of the forms $k\mathrm{e}^{px}$, $A+Bx$, $p+qx+cx^2$ or $m\cos\omega x+n\sin\omega x$
	9.6	Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation.	

First order Differential Equations

A differential equation is an equation involving a derivative. A 'first order' differential equation means the equation contains the first derivative ($\frac{dy}{dx}$) but not the second derivative or beyond. D.E's are used to model situations which involve rates of change and their solution gives the relationship between the variables themselves, not their derivatives.

- General Solution:

- Particular Solution:

Solving First Order DE's

There are 3 methods to solve first order DE's:

1. Separating variables
2. Reverse Product rule – perfect derivative
3. Integrating Factor to produce a perfect derivative

1. Separating Variables (Pure Year 2 Recap)

Examples:

1. $\frac{dy}{dx} = 2$

2. Find general solutions to $\frac{dy}{dx} = -\frac{x}{y}$

3. Find general solutions to $\frac{dy}{dx} = xy + x$

4. Find general solutions to $\frac{dy}{dx} = -\frac{y}{x}$

2. Reverse Product Rule

How could we find general solutions of the equation $x^3 \frac{dy}{dx} + 3x^2 y = \sin x$

We can't separate the variables. But do you notice anything about the LHS?

Examples

1. $x \frac{dy}{dx} + y = cosx$

2. $x^2 e^y \frac{dy}{dx} + 2xe^y = x$

Test Your Understanding

1. Find general solutions of the equation $\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = e^x$

2. Find general solutions of the equation $4xy\frac{dy}{dx} + 2y^2 = x^2$