7.1) Hypothesis testing

Worked example

Joan believes a six-sided dice is biased in favour of rolling a 4.
She rolls the dice 10 times and counts the number of times, X, it rolls a 4.
Define the test statistic and state your null and alternative hypotheses.

Your turn

John believes a coin is biased in favour of landing with tails uppermost.
He tosses the coin 8 times and counts the number of times, X, it lands with tails uppermost.
Define the test statistic and state your null and alternative hypotheses.

$X =$ number of tosses that land on tails
$p =$ probability/proportion of tosses that land on tails

$H_0: p = 0.5$
$H_1: p > 0.5$

Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether the candidate is over-estimating his support.
The researcher asks 30 people whether they support the candidate or not. 2 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.
A researcher wants to test, at the 5% significance level, whether the candidate is over-estimating her support.
The researcher asks 20 people whether they support the candidate or not. 3 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

a) X = number of people who say they support the candidate

b) p = probability/proportion of people that support the candidate
$H_0: p = 0.4$
$H_1: p < 0.4$

c) Reject H_0 if $P(X \leq 3) < 0.05$

Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 1% significance level, whether the candidate is under-estimating his support.
The researcher asks 30 people whether they support the candidate or not. 11 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.
A researcher wants to test, at the 2% significance level, whether the candidate is under-estimating her support.
The researcher asks 20 people whether they support the candidate or not. 12 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

a) X = number of people who say they support the candidate

b) p = probability/proportion of people that support the candidate

$H_0: p = 0.4$
$H_1: p > 0.4$

c) Reject H_0 if $P(X \geq 12) < 0.05$

Worked example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
A sample of 200 lightbulbs is tested, and 11 are found to be faulty.
The manager wishes to test at the 2% significance level whether or not there has been a reduction in the proportion of faulty lightbulbs.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Your turn

In a manufacturing process, the proportion of faulty bolts is, based on historical data, 0.07.
A sample of 100 bolts is tested, and 4 are found to be faulty.
The manager wishes to test at the 1% significance level whether or not there has been a reduction in the proportion of faulty bolts.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

a) X = number of faulty bolts

b) p = probability/proportion of faulty bolts

$H_0: p = 0.07$
$H_1: p < 0.07$

c) Reject H_0 if $P(X \leq 4) < 0.01$

Worked example

Joan believes the probability of rolling a 4 on a six-sided dice is $\frac{1}{6}$.
She rolls the dice 10 times and counts the number of times, X, it rolls a 4.
Define the test statistic and state your null and alternative hypotheses.

Your turn

John believes a coin is lands on tails with probability $\frac{1}{2}$.
He tosses the coin 8 times and counts the number of times, X, it lands with tails uppermost.
Define the test statistic and state your null and alternative hypotheses.

$X =$ number of tosses that land on tails
$p =$ probability/proportion of tosses that land on tails

$H_0: p = 0.5$
$H_1: p \neq 0.5$

Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 10% significance level, whether this claim is true.
The researcher asks 30 people whether they support the candidate or not. 2 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.
A researcher wants to test, at the 5% significance level, whether this claim is true.
The researcher asks 20 people whether they support the candidate or not. 3 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

a) X = number of people who say they support the candidate

b) p = probability/proportion of people that support the candidate

$H_0: p = 0.4$
$H_1: p \neq 0.4$

c) Reject H_0 if $P(X \leq 3) < 0.025$

Worked example

An election candidate believes he has the support of 30% of the residents in a particular town.
A researcher wants to test, at the 1% significance level, whether this claim is true.
The researcher asks 30 people whether they support the candidate or not. 11 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Your turn

An election candidate believes she has the support of 40% of the residents in a particular town.
A researcher wants to test, at the 2% significance level, whether this claim is true.
The researcher asks 20 people whether they support the candidate or not. 12 people say they do.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

a) X = number of people who say they support the candidate

b) p = probability/proportion of people that support the candidate

$H_0: p = 0.4$

$H_1: p \neq 0.4$

c) Reject H_0 if $P(X \geq 12) < 0.01$

Worked example

In a manufacturing process, the proportion of faulty lightbulbs is, based on historical data, 0.08.
The manufacturing process is changed.
A sample of 200 lightbulbs is tested, and 11 are found to be faulty.
The manager wishes to test at the 2% significance level whether or not there has been a change in the proportion of faulty lightbulbs.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

Your turn

In a manufacturing process, the proportion of faulty bolts is, based on historical data, 0.07.
The manufacturing process is changed.
A sample of 100 bolts is tested, and 4 are found to be faulty.
The manager wishes to test at the 1% significance level whether or not there has been a change in the proportion of faulty bolts.

a) Write down a suitable test statistic.
b) Write down two suitable hypotheses.
c) Explain the condition under which the null hypothesis would be rejected.

a) X = number of faulty bolts

b) p = probability/proportion of faulty bolts

$H_0: p = 0.07$
$H_1: p \neq 0.07$

c) Reject H_0 if $P(X \leq 4) < 0.005$