

## 7.1) Addition formulae

## Worked example

Given that

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

find  $\sin(A - B)$

Given that

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

find  $\cos(A - B)$

## Your turn

Given that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

find  $\tan(A - B)$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Worked example

Use the formulae for  $\sin(A - B)$  and  $\cos(A - B)$  to prove that

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Your turn

Use the formulae for  $\sin(A + B)$  and  $\cos(A + B)$  to prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

**Proof**

## Worked example

Given that  $3 \sin(x - y) = 2 \cos(x + y)$   
express  $\tan x$  in terms of  $\tan y$ .

## Your turn

Given that  $2 \sin(x + y) = 3 \cos(x - y)$   
express  $\tan x$  in terms of  $\tan y$ .

$$\tan x = \frac{3 - 2 \tan y}{2 - 3 \tan y}$$

## Worked example

Express the following as a single sine, cosine or tangent, and evaluate:

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

$$\cos 20^\circ \cos 25^\circ - \sin 20^\circ \sin 25^\circ$$

$$\frac{\tan \frac{\pi}{18} + \tan \frac{\pi}{9}}{1 - \tan \frac{\pi}{18} \tan \frac{\pi}{9}}$$

## Your turn

Express the following as a single sine, cosine or tangent, and evaluate:

$$\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 70^\circ \cos 25^\circ + \sin 70^\circ \sin 25^\circ$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\frac{\tan \frac{13\pi}{36} - \tan \frac{\pi}{9}}{1 + \tan \frac{13\pi}{36} \tan \frac{\pi}{9}}$$

$$\tan \frac{\pi}{4} = 1$$

## Worked example

Write in the form  $\sin(x \pm \theta)$  or  $\cos(x \pm \theta)$  where  $0 < \theta < \frac{\pi}{2}$ :

$$\frac{1}{2}(\sqrt{3} \sin x + \cos x)$$

$$\frac{1}{2}(\sqrt{3} \cos x - \sin x)$$

## Your turn

Write in the form  $\sin(x \pm \theta)$  or  $\cos(x \pm \theta)$  where  $0 < \theta < \frac{\pi}{2}$ :

$$\frac{1}{2}(\sin x - \sqrt{3} \cos x)$$

$$\sin\left(x + \frac{\pi}{3}\right)$$

## Worked example

Given that  $\tan(x + \frac{\pi}{6}) = \frac{1}{2}$  evaluate  $\tan x$

Given that  $\tan(x - \frac{\pi}{3}) = \frac{1}{2}$  evaluate  $\tan x$

## Your turn

Given that  $\tan(x + \frac{\pi}{4}) = \frac{1}{2}$  evaluate  $\tan x$

$$\tan x = -\frac{1}{3}$$