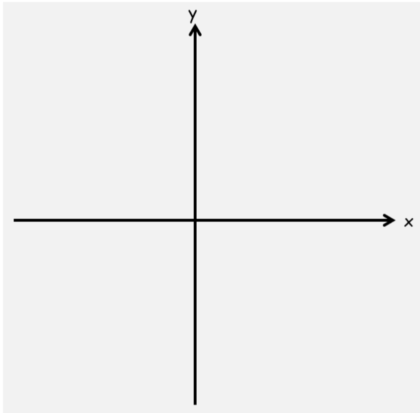
7A Part 1 First Order Differential Equations 2.0

1. Find the general solution of the differential equation, then sketch members of the family of solution curves represented by the general solution.

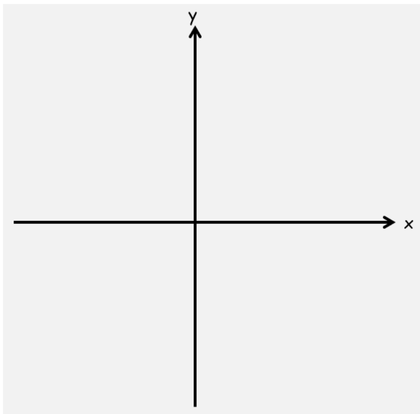
a)

$$\frac{dy}{dx} = 2$$



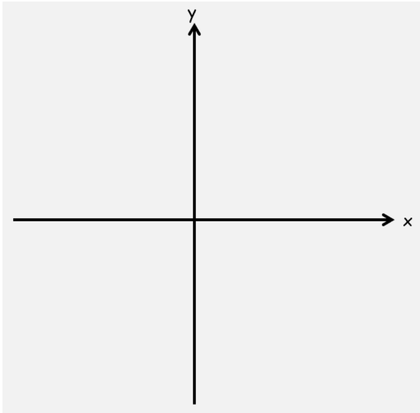
b)

$$\frac{dy}{dx} = -\frac{x}{y}$$



c)

$$\frac{dy}{dx} = -\frac{y}{x}$$



Product Rule Examples (The new stuff)

2. Find the general solution of the following equation:

a)

$$x^3\frac{dy}{dx} + 3x^2y = sinx$$

b)

$$6x^2y\frac{dy}{dx} + 6xy^2 = sec^2x$$

c)

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{5}{4x^2}$$

7A Part 2 First Order Differential Equations and Integrating Factors

Notes:

Solve the general equation:

$$\frac{dy}{dx} + Py = Q$$

Where P and Q are functions of x.

1. Find the general solution of the equation:

$$\frac{dy}{dx} - 4y = e^{x}$$

2. Find the general solution of the equation:

$$\cos x \frac{dy}{dx} + 2y \sin x = cos^{4} x$$

7B Homogeneous Second Order Differential Equations

$$\boldsymbol{a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0}$$

Solve the equation:

$$a\frac{dy}{dx} + by = 0$$

Case 1: $b^2 > 4ac$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when

$$y = Ae^{mx}$$

Find the auxiliary equation for

$$\boldsymbol{a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0}$$

Summary: when $b^2 > 4ac$ then the solution will be in the form…

1. Find the general solution of the equation:

$$\frac{d^2y}{dx^2}-7\frac{dy}{dx}+12y=0$$

Case 2: $b^2 = 4ac$

Show that

$$y = (A + Bx)e^{3x}$$

Satisfies the equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

Summary: If $b^2=4ac$ then the solution will be in the form…

2. Find the general solution of the equation:

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$$

Case 3: $b^2 < 4ac$

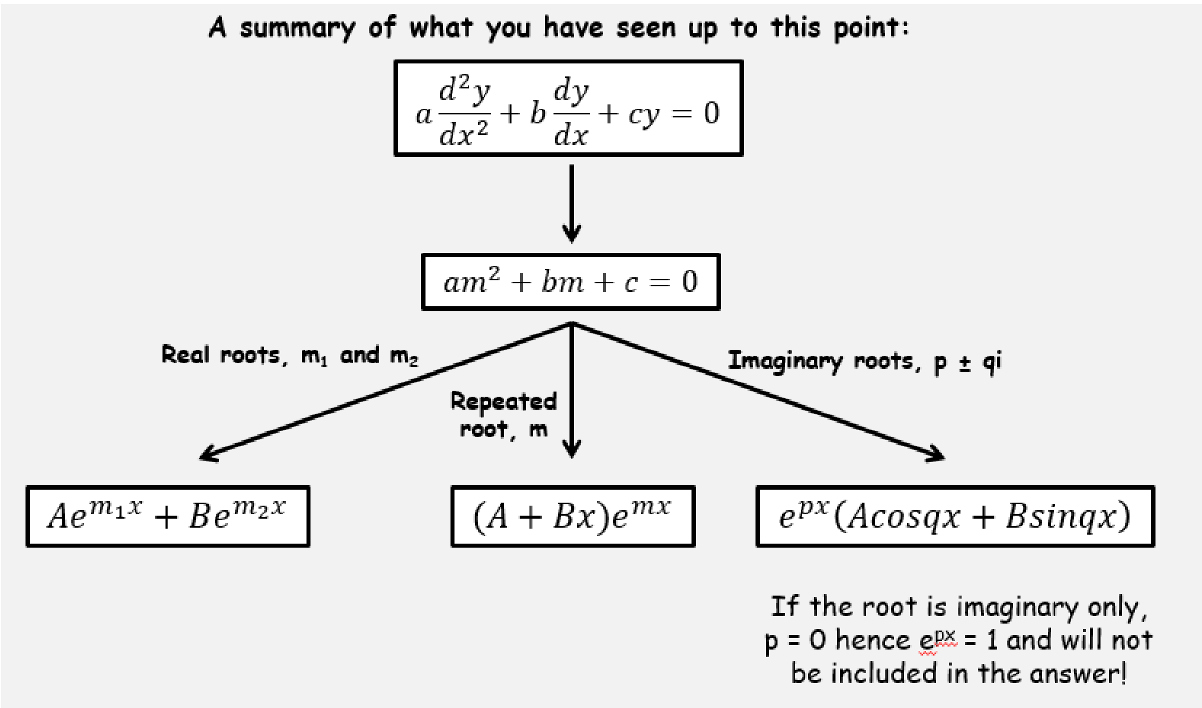
Find the general solution of the differential equation:

$$\frac{d^2y}{dx^2} + 16y = 0$$

3. Find the general solution of the differential equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 34y = 0$$

Summary: If $b^2=4ac$ then the solution will be in the form…



7C Particular Integrals of Second Order Differential Equations

$$\boldsymbol{a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)}$$

1. Find the solution of the differential equation:

a)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3$$

b)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2x$$

c)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3x^2$$

d)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x$$

e)

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 13sin3x$$

2. Find the general solution to the following differential equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3$$

Summary:

1) Start by finding the Complimentary Function by setting the differential equation equal to 0, then forming the auxiliary equation (as in the previous sections)

2) Find the Particular Integral by considering f(x) and letting y equal something of the same form. Then differentiate it and replace these in the original equation and solve for the unknowns

→ Use the table to the right (which you are NOT given...)

3) Combine the CF and PI to create the equation in y

Form of f(x)	Form of PI
k	λ
kx	λx + μ
kx^2	$λx^2$ + μx + v
ke^{px}	$λe^{px}$
mcosax	λcosax + μsinax
msinax	λcosax + μsinax
mcosax+nsinax	λcosax + μsinax

If the form of the Particular Integral is already in the Complimentary Function, include an 'x' in it as well (as we did on the last example!)

7D Finding Constants of Second Order Differential Equations

1. Find y in terms of x, given that:

$$\frac{d^2y}{dx^2} - y = 2e^x$$

   And that when x = 0,

$$y = 0 \ \ and \ \ \frac{dy}{dx} = 0$$

2. Given that the particular integral is of the form:

$$\lambda sin2t$$

Find the solution of the differential equation:

$$\frac{d^2x}{dt^2} + x = 3sin2t$$

When t = 0, x = 0 and $^{dx}/_{dt}$ = 1