7) Trigonometry and modelling

7.1) Addition formulae

Worked example	Your turn
Given that $sin(A+B) = sin\,A\,cos\,B + cos\,A\,sin\,B$ find $\sin(A-B)$	Given that $$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$ find $tan(A-B)$ $$tan(A-B) = \frac{tan\,A - tan\,B}{1 + tan\,A\,tan\,B}$$
Given that $\cos(A+B) = \cos A \cos B - \sin A \sin B$ find $cos(A-B)$	

Worked example	Your turn
Use the formulae for $\sin(A-B)$ and $\cos(A-B)$ to prove that $$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$	Use the formulae for $\sin(A+B)$ and $\cos(A+B)$ to prove that $$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$ Proof

Worked example	Your turn
Given that $3\sin(x-y) = 2\cos(x+y)$ express $\tan x$ in terms of $\tan y$.	Given that $2\sin(x+y) = 3\cos(x-y)$ express $\tan x$ in terms of $\tan y$. $\tan x = \dfrac{3 - 2\tan y}{2 - 3\tan y}$

Worked example	Your turn
Express the following as a single sine, cosine or tangent, and evaluate: $\sin 30° \cos 60° + \cos 30° \sin 60°$	Express the following as a single sine, cosine or tangent, and evaluate: $\sin 60° \cos 30° - \cos 60° \sin 30°$ $\sin 30° = \frac{1}{2}$
$\cos 20° \cos 25° - \sin 20° \sin 25°$	$\cos 70° \cos 25° + \sin 70° \sin 25°$ $\cos 45° = \frac{\sqrt{2}}{2}$
$\dfrac{\tan\frac{\pi}{18} + \tan\frac{\pi}{9}}{1 - \tan\frac{\pi}{18}\tan\frac{\pi}{9}}$	$\dfrac{\tan\frac{13\pi}{36} - \tan\frac{\pi}{9}}{1 + \tan\frac{13\pi}{36}\tan\frac{\pi}{9}}$ $\tan\frac{\pi}{4} = 1$

Worked example	Your turn
Write in the form $\sin(x \pm \theta)$ or $\cos(x \pm \theta)$ where $0 < \theta < \frac{\pi}{2}$:	Write in the form $\sin(x \pm \theta)$ or $\cos(x \pm \theta)$ where $0 < \theta < \frac{\pi}{2}$:
$$\frac{1}{2}(\sqrt{3}\sin x + \cos x)$$	$$\frac{1}{2}(\sin x - \sqrt{3}\cos x)$$
	$$\sin(x + \frac{\pi}{3})$$
$$\frac{1}{2}(\sqrt{3}\cos x - \sin x)$$	

Worked example	Your turn
Given that $\tan(x + \frac{\pi}{6}) = \frac{1}{2}$ evaluate $\tan x$	Given that $\tan(x + \frac{\pi}{4}) = \frac{1}{2}$ evaluate $\tan x$ $\tan x = -\frac{1}{3}$
Given that $\tan(x - \frac{\pi}{3}) = \frac{1}{2}$ evaluate $\tan x$	

7.2) Using the angle addition formulae

Worked example	Your turn
Using the trigonometric angle addition formulae find: $\sin 75°$ $\tan 75°$	Using the trigonometric angle addition formulae find: $\cos 75°$ $\frac{\sqrt{6}-\sqrt{2}}{4}$

Worked example	Your turn
Using the trigonometric angle addition formulae find: $\sin 15°$ $\tan 15°$	Using the trigonometric angle addition formulae find: $\cos 15°$ $\frac{\sqrt{6}+\sqrt{2}}{4}$

Worked example	Your turn

Worked example

Using the trigonometric angle addition formulae evaluate:

$$\sin 30° \cos 60° + \cos 30° \sin 60°$$

$$\cos 20° \cos 25° - \sin 20° \sin 25°$$

$$\frac{\tan\frac{\pi}{18} + \tan\frac{\pi}{9}}{1 - \tan\frac{\pi}{18}\tan\frac{\pi}{9}}$$

Your turn

Using the trigonometric angle addition formulae evaluate:

$$\sin 60° \cos 30° - \cos 60° \sin 30°$$

$$\sin 30° = \frac{1}{2}$$

$$\cos 70° \cos 25° + \sin 70° \sin 25°$$

$$\cos 45° = \frac{\sqrt{2}}{2}$$

$$\frac{\tan\frac{13\pi}{36} - \tan\frac{\pi}{9}}{1 + \tan\frac{13\pi}{36}\tan\frac{\pi}{9}}$$

$$\tan\frac{\pi}{4} = 1$$

Worked example	Your turn
Given that: $\sin A = \frac{8}{17}$ and $0° < A < 90°$, and $\cos B = -\frac{4}{5}$, B is obtuse, find the value of $\cos(A + B)$	Given that: $\sin A = -\frac{3}{5}$ and $180° < A < 270°$, and $\cos B = -\frac{12}{13}$, B is obtuse, find the value of $\cos(A - B)$ $\frac{33}{65}$

Worked example	Your turn
Given that: $\sin A = \frac{8}{17}$ and $0° < A < 90°$, and $\cos B = -\frac{4}{5}$, B is obtuse, find the value of $\tan(A - B)$	Given that: $\sin A = -\frac{3}{5}$ and $180° < A < 270°$, and $\cos B = -\frac{12}{13}$, B is obtuse, find the value of $\tan(A + B)$ $\frac{16}{63}$

Worked example	Your turn
Given that: $\sin A = \frac{8}{17}$ and $0° < A < 90°$, and $\cos B = -\frac{4}{5}$, B is obtuse, find the value of $\sec(A - B)$	Given that: $\sin A = -\frac{3}{5}$ and $180° < A < 270°$, and $\cos B = -\frac{12}{13}$, B is obtuse, find the value of $\text{cosec}(A + B)$ $\frac{65}{16}$

Worked example	Your turn
Given that $2\cos(x-40)° = \sin(x-50)°$ show that $\tan x = 3\tan 50°$	Given that $2\cos(x+50)° = \sin(x+40)°$ show that $\tan x = \frac{1}{3}\tan 40°$ Shown

7.3) Double-angle formulae

Worked example	Your turn
Using the trigonometric angle addition formulae, derive: $\sin 2x$ $\tan 2x$	Using the trigonometric angle addition formulae find: $\cos 2x$ $\cos^2 x - \sin^2 x$

Worked example	Your turn
Using $cos2x \equiv \cos^2 x - \sin^2 x$, express $\cos 2x$ using only terms of $\cos^2 x$ and constants	Using $cos2x \equiv \cos^2 x - \sin^2 x$, express $\cos 2x$ using only terms of $\sin^2 x$ and constants $1 - 2\sin^2 x$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio:	Use the double-angle formulae to write as a single trigonometric ratio:
$\cos^2 50° - \sin^2 50°$	$\cos^2 15° - \sin^2 15°$
	$\cos 30° = \frac{\sqrt{3}}{2}$
$2\cos^2 \frac{2\pi}{9} - 1$	$2\cos^2 22.5° - 1$
	$\cos 45° = \frac{\sqrt{2}}{2}$
$1 - 2\sin^2 30°$	$1 - 2\sin^2 \frac{\pi}{4}$
	$\cos \frac{\pi}{2} = 0$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio: $\cos^2 2x - \sin^2 2x$	Use the double-angle formulae to write as a single trigonometric ratio: $\cos^2 5x - \sin^2 5x$ $\cos 10x$
$4\cos^2 3x - 2$	$8\cos^2 6x - 4$ $4\cos 12x$
$3 - 6\sin^2 4x$	$5 - 10\sin^2 7x$ $5\cos 14x$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio:	Use the double-angle formulae to write as a single trigonometric ratio:
$2\sin 45° \cos 45°$	$2\sin 30° \cos 30°$ $\sin 60° = \frac{\sqrt{3}}{2}$
$4\sin\frac{\pi}{12}\cos\frac{\pi}{12}$	$8\sin\frac{\pi}{4}\cos\frac{\pi}{4}$ $16\sin\frac{\pi}{2} = 16$
$7\sin 5x \cos 5x$	$5\sin 7x \cos 7x$ $10\sin 14x$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio:	Use the double-angle formulae to write as a single trigonometric ratio:
$$\frac{2\tan 30°}{1-\tan^2 30°}$$	$$\frac{2\tan 22.5°}{1-\tan^2 22.5°}$$ $\tan 45° = 1$
$$\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$$	$$\frac{2\tan\frac{\pi}{4}}{1-\tan^2\frac{\pi}{4}}$$ 0
$$\frac{4\tan 6x}{1-\tan^2 6x}$$	$$\frac{6\tan 4x}{1-\tan^2 4x}$$ $3\tan 8x$

Worked example	Your turn
Use the double-angle formulae to write as a single trigonometric ratio: $\frac{8\sin 22.5°}{\sec 22.5°}$ $\frac{6\cos\frac{\pi}{4}}{\mathrm{cosec}\frac{\pi}{4}}$	Use the double-angle formulae to write as a single trigonometric ratio: $\frac{4\sin 30°}{\sec 30°}$ $2\sin 60° = \sqrt{3}$

Worked example	Your turn
Given that $x = 2\sin\theta$ and $y = 4 - 3cos2\theta$, eliminate θ and express y in terms of x. Given that $x = 5\cos\theta$ and $y = 6 - 7cos2\theta$, eliminate θ and express y in terms of x.	Given that $x = 3\sin\theta$ and $y = 3 - 4cos2\theta$, eliminate θ and express y in terms of x. $y = \frac{8x^2}{9} - 1$

Worked example	Your turn
Given that $\cos x = \frac{5}{8}$ and x is acute, find the exact value of (a) $\sin 2x$ (b) $\tan 2x$	Given that $\cos x = \frac{3}{4}$ and x is acute, find the exact value of (a) $\sin 2x$ (b) $\tan 2x$ (a) $\frac{3\sqrt{7}}{8}$ (b) $3\sqrt{7}$

Worked example	Your turn
Using the double-angle formulae, evaluate: $\left(\sin\frac{\pi}{3}+\cos\frac{\pi}{3}\right)^2$ $\left(\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\right)^2$	Using the double-angle formulae, evaluate: $\left(\sin\frac{\pi}{6}+\cos\frac{\pi}{6}\right)^2$ $\frac{2+\sqrt{3}}{2}$

Worked example	Your turn
Given that $0 < \theta < \pi$, find the value of $\tan\frac{\theta}{2}$ when $\tan\theta = -\frac{3}{4}$	Given that $\pi < \theta < \frac{3\pi}{2}$, find the value of $\tan\frac{\theta}{2}$ when $\tan\theta = \frac{3}{4}$ -3

7.4) Solving trigonometric equations

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$8\sin(\theta + 60°) = 4\sqrt{2}\cos\theta$$	Solve in the interval $0 \leq x \leq 360°$: $$4\sin(\theta + 30°) = 8\sqrt{2}\cos\theta$$ $\theta = 69.6°, 249.6°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$8\cos(\theta - 60°) = 4\sqrt{2}\sin\theta$$	Solve in the interval $0 \leq x \leq 360°$: $$4\cos(\theta - 30°) = 8\sqrt{2}\sin\theta$$ $\theta = 20.4°, 200.4°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $3\cos 2x + \cos x + 2 = 0$	Solve in the interval $0 \leq x \leq 360°$: $3\cos 2x - \cos x + 2 = 0$ $x = 60.0°, 109.5°, 250.5°, 300.0°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$3\cos 2x - \sin x - 2 = 0$$	Solve in the interval $0 \leq x \leq 360°$: $$3\cos 2x + \sin x - 2 = 0$$ $x = 30.0°, 150.0°, 199.5°, 340.5°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $5\sin 2x + 4\sin x = 0$ $4\sin 2x - 5\cos x = 0$	Solve in the interval $0 \leq x \leq 360°$: $5\sin 2x - 4\sin x = 0$ $x = 0.0°, 66.4°, 180.0°, 199.5°, 293.6°, 360.0°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq y \leq 2\pi$: $$3\tan 2y \tan y = 2$$	Solve in the interval $0 \leq y \leq 2\pi$: $$2\tan 2y \tan y = 3$$ $y = 0.58, 2.56, 3.72, 5.70$ (2 dp)

Worked example	Your turn
a) Show that $\cos(3A) = 4\cos^3 A - 3\cos A$. b) Hence or otherwise, solve, for $0 < \theta < 2\pi$, the equation $12\cos\theta - 16\cos^3\theta - 2\sqrt{3} = 0$	a) Show that $\sin(3A) = 3\sin A - 4\sin^3 A$. b) Hence or otherwise, solve, for $0 < \theta < 2\pi$, the equation $16\sin^3\theta - 12\sin\theta - 2\sqrt{3} = 0$ a) Shown b) $\theta = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

7.5) Simplifying $a \cos x \pm b \sin x$

Worked example	Your turn
Express $5 \sin x + 12 \cos x$ in the form: $R \sin(x + \alpha), R > 0, 0 < \alpha < 90°$	Express $3 \sin x + 4 \cos x$ in the form: $R \sin(x + \alpha), R > 0, 0 < \alpha < 90°$ $5 \sin(x + 53.1°)$

Worked example	Your turn
Express $5\sin x + 12\cos x$ in the form: $R\cos(x-\alpha), R>0, 0<\alpha<90°$	Express $3\sin x + 4\cos x$ in the form: $R\cos(x-\alpha), R>0, 0<\alpha<90°$ $5\cos(x-36.9°)$

Worked example	Your turn
Express $5\sin x + 12\cos x$ in the form: $R\sin(x-\alpha), R>0, 0<\alpha<180°$	Express $3\sin x + 4\cos x$ in the form: $R\sin(x-\alpha), R>0, 0<\alpha<180°$ $5\sin(x+126.9°)$

Worked example	Your turn
Express $5 \sin x + 12 \cos x$ in the form: $R\cos(x+\alpha), R > 0, 0 < \alpha < 180°$	Express $3 \sin x + 4 \cos x$ in the form: $R\cos(x+\alpha), R > 0, 0 < \alpha < 180°$ $5\cos(x + 143.1°)$

Worked example	Your turn
Solve in the interval $0 < \theta < 360°$: $5\cos\theta + 2\sin\theta = 3$	Solve in the interval $0 < \theta < 360°$: $2\cos\theta + 5\sin\theta = 3$ $\theta = 12.1°, 124.3°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq \theta < 180°$: $5\sin 3\theta - 12\cos 3\theta = 1$	Solve in the interval $0 \leq \theta < 180°$: $3\sin 3\theta - 4\cos 3\theta = 1$ $\theta = 21.6°, 73.9°, 141.6°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq \theta < 180°$: $5\cos 3\theta - 12\sin 3\theta = 1$	Solve in the interval $0 \leq \theta < 180°$: $3\cos 3\theta - 4\sin 3\theta = 1$ $\theta = 8.4°, 76.1°, 128.4°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq \theta < 360°$: $\cot\theta + 4 = cosec\ \theta$	Solve in the interval $0 \leq \theta < 360°$: $\cot\theta + 3 = cosec\ \theta$ $\theta = 143.1°$ (1 dp)

Worked example	Your turn
Find the maximum value and the smallest positive value of θ at which the maximum occurs for: $$4\cos\theta + 3\sin\theta$$	Find the maximum value and the smallest positive value of θ at which the maximum occurs for: $$12\cos\theta + 5\sin\theta$$ Maximum $= 13$ when $\theta = 22.6°$ (1 dp)

Worked example	Your turn
Find the minimum value and the smallest positive value of θ at which the minimum occurs for: $$4\cos\theta + 3\sin\theta$$	Find the minimum value and the smallest positive value of θ at which the minimum occurs for: $$12\cos\theta + 5\sin\theta$$ Minimum $= -13$ when $\theta = 202.6°$ (1 dp)

Worked example	Your turn
Find the maximum value and the smallest positive value of θ at which the maximum occurs for: $$\frac{5}{7+4\cos\theta-3\sin\theta}$$	Find the maximum value and the smallest positive value of θ at which the maximum occurs for: $$\frac{3}{17+12\cos\theta-5\sin\theta}$$ Maximum $=\frac{3}{4}$ when $\theta = 157.4°$ (1 dp)

Worked example	Your turn
Find the minimum value and the smallest positive value of θ at which the minimum occurs for: $$\frac{5}{7+4\cos\theta-3\sin\theta}$$	Find the minimum value and the smallest positive value of θ at which the minimum occurs for: $$\frac{3}{17+12\cos\theta-5\sin\theta}$$ Minimum $=\frac{1}{10}$ when $\theta=337.4°$ (1 dp)

7.6) Proving trigonometric identities

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Worked example

Prove that:

$$\cot 2\theta \equiv \frac{\cot\theta - \tan\theta}{2}$$

Your turn

Prove that:

$$\tan 2\theta \equiv \frac{2}{\cot\theta - \tan\theta}$$

Proof

Worked example	Your turn
Prove that: $$\frac{-\sin 2\theta}{\cos 2\theta - 1} \equiv \cot\theta$$	Prove that: $$\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta$$ Proof

Worked example	Your turn
Prove that: $\cot 2x - cosec\ 2x \equiv -\tan x$	Prove that: $\cot 2x + cosec\ 2x \equiv \cot x$ Proof

Worked example	Your turn
Prove, starting with the left-hand side: $\tan 2x + sec\ 2x \equiv \frac{\cos x + \sin x}{\cos x - \sin x}$	Prove, starting with the right-hand side: $\tan 2x + sec\ 2x \equiv \frac{\cos x + \sin x}{\cos x - \sin x}$ Proof

Worked example	Your turn
Show that: $$\sin^4\theta = \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$$	Show that: $$\cos^4\theta = \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$$ Shown

Worked example	Your turn
By writing $\cos x = \cos\left(2 \times \frac{x}{2}\right)$, prove the identity $$\frac{1 + \cos x}{1 - \cos x} \equiv \cot^2\left(\frac{x}{2}\right)$$	By writing $\cos x = \cos\left(2 \times \frac{x}{2}\right)$, prove the identity $$\frac{1 - \cos x}{1 + \cos x} \equiv \tan^2\left(\frac{x}{2}\right)$$ Proof

7.7) Modelling with trigonometric functions

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Worked example

The cabin pressure, P (psi) on an aeroplane at cruising altitude can be modelled by the equation

$$P = 14.5 - 0.2\sin(t - 3)$$

where t is the time in hours since cruising altitude was first reached, and angles are in radians. Find:

a) The maximum and minimum cabin pressure
b) The time after reaching cruising altitude that the cabin first reaches a maximum pressure
c) The cabin pressure after 3 hours at cruising altitude
d) All the times within the first 10 hours of cruising that the cabin pressure would be exactly 14.42 psi

Your turn

The cabin pressure, P (psi) on an aeroplane at cruising altitude can be modelled by the equation

$$P = 11.5 - 0.5\sin(t - 2)$$

where t is the time in hours since cruising altitude was first reached, and angles are in radians. Find:

a) The maximum and minimum cabin pressure
b) The time after reaching cruising altitude that the cabin first reaches a maximum pressure
c) The cabin pressure after 5 hours at cruising altitude
d) All the times within the first 8 hours of cruising that the cabin pressure would be exactly 11.3 psi

a) Maximum = 12 psi ; Minimum = 11 psi
b) 0.43 hours = 26 minutes
c) 11.43 psi
d) 2 hours 25 minutes, and 4 hours 44 minutes

Worked example

a) Express $7\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $7\cos\theta - 5\sin\theta$ and the value of θ, for $0 < \theta < 2\pi$ at which this maximum occurs.

The height H above ground of a passenger on a Ferris wheel is modelled by the equation

$$H = 12 - 7\cos\left(\frac{\pi t}{4}\right) + 5\sin\left(\frac{\pi t}{4}\right)$$

where H is measured in metres, and t is the time in minutes after the wheel starts turning.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum first occurs

d) Determine the time for the Ferris wheel to complete five revolutions

Your turn

a) Express $9\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $9\cos\theta - 2\sin\theta$ and the value of θ, for $0 < \theta < 2\pi$ at which this maximum occurs.

The height H above ground of a passenger on a Ferris wheel is modelled by the equation

$$H = 10 - 9\cos\left(\frac{\pi t}{5}\right) + 2\sin\left(\frac{\pi t}{5}\right)$$

where H is measured in metres, and t is the time in minutes after the wheel starts turning.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum first occurs

d) Determine the time for the Ferris wheel to complete two revolutions

a) $R = \sqrt{85}, \alpha = 0.2187$

b) Maximum $= \sqrt{85}$ when $\theta = 6.06$

c) Maximum $H = 19.22m$ at $t = 4.65$

d) 20 minutes

Worked example

a) Express $1.5\sin\theta - 2\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $1.5\sin\theta - 2\cos\theta$ and the value of θ, for $0<\theta<\pi$ at which this maximum occurs.

The height H of sea water on a particular day can be modelled by the equation

$$H = 8 + 1.5\sin\left(\frac{2\pi t}{25}\right) - 2\cos\left(\frac{2\pi t}{25}\right), 0 \le t < 12$$

where H is measured in metres, and t is the number of hours after midnight.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum occurs

d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

Your turn

a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. State the exact value of R and give α to four decimal places.

b) State the maximum value of $2\sin\theta - 1.5\cos\theta$ and the value of θ, for $0<\theta<\pi$ at which this maximum occurs.

The height H of sea water on a particular day can be modelled by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), 0 \le t < 12$$

where H is measured in metres, and t is the number of hours after midday.

c) Calculate the maximum value of H predicted by this model, and the value of t when this maximum occurs

d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

a) $R = 2.5, \alpha = 0.6435$

b) Maximum $= 2.5$ when $\theta = 2.21$

c) Maximum $H = 8.5m$ at $t = 4.41$

d) $14{:}06$ and $18{:}43$