7) Algebraic methods

7.1) Algebraic fractions
7.2) Dividing polynomials
7.3) The factor theorem

7.1) Algebraic fractions

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Worked example

Simplify:

$\frac{x^3}{x}$

$\frac{x^4}{x}$

$\frac{x^4}{x^2}$

$\frac{6x^4}{x^2}$

$\frac{6x^5}{2x^2}$

$\frac{6x^5}{8x^2}$

$\frac{6x^5}{12x^2}$

$\frac{6x^5}{18x^8}$

Your turn

Simplify:

$\frac{8x^7}{12x^3}$

$\frac{2x^4}{3}$

$\frac{9x^5}{15x^9}$

$\frac{3}{5x^4}$

Worked example	Your turn
Simplify: $\dfrac{x^2+x-6}{9-x^2}$	Simplify: $\dfrac{4-x^2}{x^2+2x-8}$ $-\dfrac{x+2}{x+4}$

Worked example

Simplify:

$$\frac{3x^2 \times 5y^3 \times 4x \times 2y^5}{10xy}$$

$$\frac{2x^3 \times 3y^5 \times x^4 \times 5y^2}{6x^2y^3}$$

Your turn

Simplify:

$$\frac{2a^4 \times 7b^3 \times 4a^2 \times 3b^7}{6a^3b^9}$$

$28a^3b$

Worked example	Your turn
Simplify: $\frac{5x+10}{x+2}$	Simplify: $\frac{x-3}{4x-12}$ $\frac{1}{4}$
$\frac{x-7}{5x-35}$	

Worked example	Your turn
Simplify: $\dfrac{x^2+5x+6}{x+2}$	Simplify: $\dfrac{x^2-7x+10}{3x-15}$
$\dfrac{4x-12}{x^2-5x+6}$	$\dfrac{x-2}{3}$

Worked example	Your turn
Simplify:	Simplify:
$\frac{x+3}{(x+3)^3}$	$\frac{(x-5)^2}{(x-5)^6}$ $\frac{1}{(x-5)^4}$
$\frac{(x-4)^5}{(x-4)^2}$	$\frac{(x+5)^3}{3x+15}$ $\frac{(x+5)^2}{3}$
$\frac{2x+10}{(x+5)^7}$	

Worked example	Your turn
Simplify: $\frac{3x^2 + 6x}{15x}$	Simplify: $\frac{4x^3 + 12x}{16x^2}$ $\frac{x^2 + 3}{4x}$
$\frac{5y^2 - 30y^4}{10y^3}$	

Worked example	Your turn
Simplify: $\dfrac{x^2+6x+5}{x^2+3x-10}$	Simplify: $\dfrac{x^2-6x+5}{x^2+4x-5}$ $\dfrac{x-5}{x+5}$
$\dfrac{x^2-7x-8}{x^2-1}$	

Worked example	Your turn
Simplify: $\dfrac{2x^2 - 5x - 3}{3x^2 - 11x + 6}$	Simplify: $\dfrac{3x^2 - 5x - 2}{2x^2 - 7x + 6}$ $\dfrac{3x + 1}{2x - 3}$
$\dfrac{3x^2 - x - 10}{x^2 - 4}$	

Worked example	Your turn
Simplify: $\frac{x^2-1}{x+1}$	Simplify: $\frac{2x+8}{x^2-16}$ $\frac{2}{x-4}$
$\frac{x-2}{x^2-4}$	
$\frac{x^2-9}{2x-6}$	

Worked example	Your turn
Simplify: $\frac{x+2}{x-3} \times \frac{x-3}{x-2}$	Simplify: $\frac{3x-4}{x+2} \times \frac{5x+10}{3x+4}$
$\frac{2x+3}{x-4} \times \frac{2x-8}{2x+1}$	$\frac{5(3x-4)}{3x+4}$

Worked example

Simplify:

$$\frac{x+2}{x-3} \div \frac{x-2}{x-3}$$

$$\frac{2x+3}{x-4} \div \frac{2x+1}{2x-8}$$

Your turn

Simplify:

$$\frac{3x-4}{x+2} \div \frac{3x+4}{5x+10}$$

$$\frac{5(3x-4)}{3x+4}$$

Worked example

Simplify:

$$\frac{(x+2)(3x-4)}{(x-5)(6x+7)} \times \frac{(7x+6)(x-5)}{(4x-3)(x+2)}$$

$$\frac{(x+7)(6x-5)}{(x-4)(3x+2)} \times \frac{(x+4)(3x+2)}{(6x+5)(7x)}$$

Your turn

Simplify:

$$\frac{(x+5)(2x-7)}{(x-4)(3x+1)} \times \frac{(6x+1)(x-4)}{(2x-9)(x+3)}$$

$$\frac{(x+5)(2x-7)(6x+1)}{(3x+1)(2x-9)(x+3)}$$

Worked example

Simplify:

$$\frac{3x^2 - 10x - 8}{6x^2 + 37x - 35} \div \frac{x^2 - 3x - 4}{x^2 - 49}$$

Your turn

Simplify:

$$\frac{2x^2 - 7x - 15}{3x^2 + 10x - 8} \div \frac{2x^2 + x - 3}{x^2 - 16}$$

$$\frac{(x - 5)(x - 4)}{(3x - 2)(x - 1)}$$

Worked example	Your turn
Simplify: $\frac{2x^3 - 5x^2 - 3x}{2x - 6}$	Simplify: $\frac{3x^3 - x^2 - 10x}{4x - 8}$ $\frac{x(3x + 5)}{4}$

Worked example	Your turn
Simplify: $\dfrac{2x^3 + 5x^2 - 3x}{4x^2 - 1}$	Simplify: $\dfrac{3x^3 - x^2 - 10x}{9x^2 - 25}$ $\dfrac{x(x-2)}{3x-5}$

Worked example	Your turn
Write as a single fraction: $3 + \frac{5}{2x-1}$	Write as a single fraction: $5 - \frac{3}{x+2}$ $\frac{5x+7}{x+2}$
$2 - \frac{3}{5x+4}$	

Worked example	Your turn
Write as a single fraction: $3-(x-4)\div\frac{x^2-16}{x-5}$	Write as a single simplified fraction: $5-(x-2)\div\frac{x^2-4}{x+3}$ $\frac{4x-13}{x-2}$

Worked example	Your turn
Write in the form $1 + \frac{a}{x+b}$:	Write in the form $1 + \frac{a}{x+b}$:
$\frac{x+3}{x-5}$	$\frac{x-5}{x+2}$
	$1 - \frac{7}{x+2}$
$\frac{x-2}{x+7}$	

Worked example	Your turn
Simplify: $\dfrac{\frac{3}{x}+2}{1+\frac{5}{x}}$	Simplify: $\dfrac{\frac{5}{3x}+3}{7+\frac{2}{3x}}$ $\dfrac{9x+5}{21x+2}$
$\dfrac{\frac{3}{2x}-2}{1-\frac{5}{2x}}$	

7.2) Dividing polynomials

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Worked example	Your turn
Divide $x^3 - 2x^2 - 17x + 10$ by $(x - 5)$	Divide $x^3 + 2x^2 - 17x + 6$ by $(x - 3)$ $x^2 + 5x - 2$

Worked example

$$f(x) = 18x^4 - 29x^2 + 3$$

Divide $f(x)$ by $(3x + 1)$.

Give your answer in the form

$$f(x) = (3x + 1)(ax^3 + bx^2 + cx + d)$$

Your turn

$$f(x) = 4x^4 - 17x^2 + 4$$

Divide $f(x)$ by $(2x + 1)$.

Give your answer in the form

$$f(x) = (2x + 1)(ax^3 + bx^2 + cx + d)$$

$$f(x) = (2x + 1)(2x^3 - x^2 - 8x + 4)$$

Worked example

Find the remainder when
$2x^3 + 5x^2 - 10x + 16$ is divided by $(x - 2)$

Your turn

Find the remainder when
$2x^3 - 5x^2 - 16x + 10$ is divided by $(x - 4)$

-6

Worked example	Your turn
Divide $27x^3 - 8$ by $3x - 2$	Divide $8x^3 - 1$ by $2x - 1$ $4x^2 + 2x + 1$

Worked example

$$f(x) = 6x^3 + 11x^2 - 46x + 24$$

Show that $(3x - 2)$ is a factor of $f(x)$ and hence find all the real roots of the equation $f(x) = 0$

Your turn

$$f(x) = 12x^3 - 14x^2 - 61x + 60$$

Show that $(2x - 3)$ is a factor of $f(x)$ and hence find all the real roots of the equation $f(x) = 0$

$$x = -\frac{5}{2}, x = \frac{3}{2}, x = \frac{4}{3}$$

7.3) The factor theorem

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Worked example	Your turn
Show that $(x-3)$ is a factor of x^3-2x^2-5x+6	Show that $(x-2)$ is a factor of x^3+x^2-4x-4 Shown (e.g. algebraic division or factor theorem)

Worked example	Your turn
Fully factorise $3x^3 + x^2 - 12x - 4$	Fully factorise $2x^3 + x^2 - 18x - 9$ $(x - 3)(2x + 1)(x + 3)$

Worked example

Given that $x + 2$ is a factor of $3x^4 - 4x^2 + a$, find the value of a.

Your turn

Given that $x + 1$ is a factor of $4x^4 - 3x^2 + a$, find the value of a.

$a = -1$

Worked example	Your turn
Given that $3x + 1$ is a factor of $12x^3 + ax^2 + 2$, find the value of a.	Given that $2x + 1$ is a factor of $6x^3 + ax^2 + 1$, find the value of a. $a = -1$

Worked example	Your turn
Given that $2x - 1$ is a factor of $2x^3 + 3x^2 + ax + 11$, find the value of a.	Given that $3x - 1$ is a factor of $3x^3 + 11x^2 + ax + 1$, find the value of a. $a = -7$

Worked example

Show that $(x-2)$ is a factor of

$$5x^4 - 16x^3 - 47x^2 + 130x - 24$$

and hence find all the real solutions to

$$5x^4 - 16x^3 - 47x^2 + 130x - 24 = 0$$

Your turn

Show that $(x-3)$ is a factor of

$$4x^4 + 15x^3 - 48x^2 - 109x + 30$$

and hence find all the real solutions to

$$4x^4 + 15x^3 - 48x^2 - 109x + 30 = 0$$

$$x = -5, x = -2, x = \frac{1}{3}, x = 3$$