7) Algebraic methods

7.1) Algebraic fractions
7.2) Dividing polynomials
7.3) The factor theorem

7.1) Algebraic fractions

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Your turn $\frac{8x^7}{12x^3}$ $\frac{2x^4}{3}$ $\frac{9x^5}{15x^9}$ $\frac{3}{5x^4}$

$$\frac{4-x^2}{x^2+2x-8}$$

$$-\frac{x+2}{x+4}$$

Worked example
Simplify:
$3x^2 \times 5y^3 \times 4x \times 2y^5$
$\overline{10xy}$
$2x^3 \times 3y^5 \times x^4 \times 5y^2$
$\frac{1}{6x^2y^3}$

$$\frac{x^2 - 7x + 10}{3x - 15}$$

$$\frac{x - 2}{3}$$

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$$\frac{(x-5)^2}{(x-5)^6}$$

$$\frac{1}{(x-5)^4}$$

$$\frac{(x+5)^3}{3x+15} \\ \frac{(x+5)^2}{3}$$

$$\frac{4x^3 + 12x}{16x^2}$$

$$\frac{x^2 + 3}{4x}$$

$$\frac{x^2 - 6x + 5}{x^2 + 4x - 5}$$

$$\frac{x - 5}{x + 5}$$

$$\frac{x^2 - 7x - 8}{x^2 - 1}$$

$$\frac{3x^{2} - 5x - 2}{2x^{2} - 7x + 6}$$

$$\frac{3x + 1}{2x - 3}$$

$$\frac{3x+1}{2x-3}$$

$$2x - 3$$

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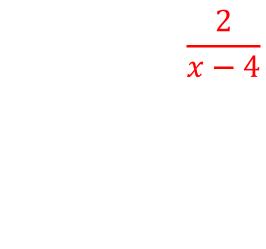
$$\frac{2x+8}{x^2-16}$$

$$\frac{2}{x-4}$$

$$\frac{x^2 - 16}{2}$$

$$\frac{2}{x - 4}$$

$$\frac{x^2 - 9}{2x - 6}$$



$$\frac{x+2}{x-3} \times \frac{x-3}{x-2}$$

$$\frac{3x - 4}{x + 2} \times \frac{5x + 10}{3x + 4}$$

$$\frac{5(3x - 4)}{3x + 4}$$

$$\frac{2x+3}{x-4} \times \frac{2x-8}{2x+1}$$

$$\frac{x+2}{x-3} \div \frac{x-2}{x-3}$$

Simplify:

$$\frac{3x - 4}{x + 2} \div \frac{3x + 4}{5x + 10}$$

$$\frac{5(3x - 4)}{3x + 4}$$

$$\frac{2x+3}{x-4} \div \frac{2x+1}{2x-8}$$

$$\frac{(x+2)(3x-4)}{(x-5)(6x+7)} \times \frac{(7x+6)(x-5)}{(4x-3)(x+2)}$$

Simplify:
$$(x + 5)$$

$$\frac{(x+5)(2x-7)}{(x-4)(3x+1)} \times \frac{(6x+1)(x-4)}{(2x-9)(x+3)}$$

$$\frac{(x+5)(2x-7)(6x+1)}{(3x+1)(2x-9)(x+3)}$$

$$\frac{(x+7)(6x-5)}{(x-4)(3x+2)} \times \frac{(x+4)(3x+2)}{(6x+5)(7x)}$$

$$\frac{3x^2 - 10x - 8}{6x^2 + 37x - 35} \div \frac{x^2 - 3x - 4}{x^2 - 49}$$

Simplify:

$$\frac{2x^{2} - 7x - 15}{3x^{2} + 10x - 8} \div \frac{2x^{2} + x - 3}{x^{2} - 16}$$
$$\frac{(x - 5)(x - 4)}{(3x - 2)(x - 1)}$$

$$\frac{3x^3 - x^2 - 10x}{4x - 8}$$

$$\frac{x(3x + 5)}{4}$$

$$\frac{3x^{3} - x^{2} - 10x}{9x^{2} - 25}$$

$$\frac{x(x - 2)}{3x - 5}$$

Write as a single fraction:

$$5 - \frac{3}{x+2}$$

$$\frac{5x+7}{x+2}$$

Write as a single fraction:

$$3 - (x - 4) \div \frac{x^2 - 16}{x - 5}$$

Write as a single simplified fraction:

$$5 - (x - 2) \div \frac{x^2 - 4}{x + 3}$$

$$\frac{4x - 13}{x - 2}$$

Write in the form $1 + \frac{a}{x+b}$: $\frac{x-5}{x+2}$ $1 - \frac{7}{x+2}$

$$\frac{x-2}{x+7}$$

$$\frac{5}{3x} + \frac{5}{3x} + \frac{9x}{21x}$$

$$\frac{\frac{3}{2x} - 2}{1 - \frac{5}{2x}}$$

$$\frac{3x}{7 + \frac{2}{3x}}$$

$$\frac{9x + 5}{21x + 2}$$

7.2) Dividing polynomials

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$$f(x) = 18x^4 - 29x^2 + 3$$

Divide $f(x)$ by $(3x + 1)$.

$$f(x) = (3x + 1)(ax^3 + bx^2 + cx + d)$$

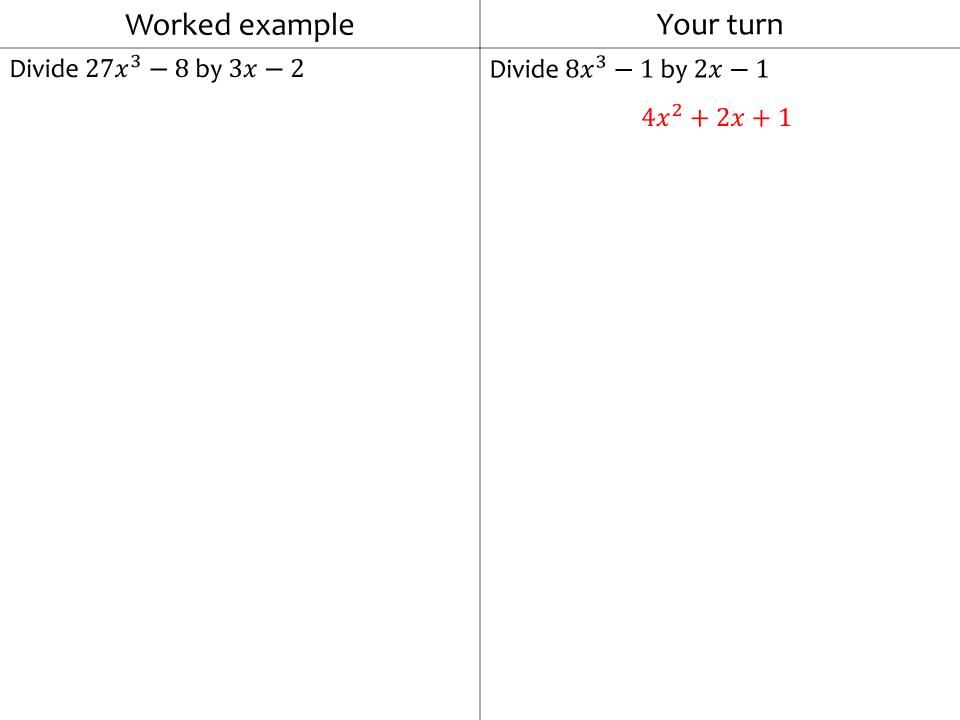
$$f(x) = 4x^4 - 17x^2 + 4$$
ivide $f(x)$ by $(2x + 1)$

Divide
$$f(x)$$
 by $(2x + 1)$.

$$f(x) = (2x+1)(ax^3 + bx^2 + cx + d)$$

$$f(x) = (2x+1)(2x^3 - x^2 - 8x + 4)$$

Worked example	Your turn
Find the remainder when $2x^3 + 5x^2 - 10x + 16$ is divided by $(x - 2)$	Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $(x - 4)$
	-6



 $f(x) = 6x^3 + 11x^2 - 46x + 24$ Show that (3x - 2) is a factor of f(x)and hence find all the real roots of the equation f(x) = 0

 $f(x) = 12x^3 - 14x^2 - 61x + 60$ Show that (2x - 3) is a factor of f(x)and hence find all the real roots of the equation f(x) = 0

$$x = -\frac{5}{2}, x = \frac{3}{2}, x = \frac{4}{3}$$

7.3) The factor theorem

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Show that $(x-3)$ is a factor of x^3-2x^2-5x+6 Show that $(x-2)$ is a factor of x^3+x^2-4x-4 Shown (e.g. algebraic division or factor theorem)	Worked example	Your turn
		$x^3 + x^2 - 4x - 4$ Shown (e.g. algebraic division or factor

Worked example	Your turn
Fully factorise $3x^3 + x^2 - 12x - 4$	Fully factorise $2x^3 + x^2 - 18x - 9$
	(x-3)(2x+1)(x+3)

Worked example	Your turn
Given that $2x - 1$ is a factor of $2x^3 + 3x^2 + ax + 11$, find the value of	Given that $3x - 1$ is a factor of $3x^3 + 11x^2 + ax + 1$, find the value of
<i>a</i> .	a. $a = -7$

Show that
$$(x - 2)$$
 is a factor of $5x^4 - 16x^3 - 47x^2 + 130x - 24$ and hence find all the real solutions to $5x^4 - 16x^3 - 47x^2 + 130x - 24 = 0$

Show that (x - 3) is a factor of $4x^4 + 15x^3 - 48x^2 - 109x + 30$ and hence find all the real solutions to $4x^4 + 15x^3 - 48x^2 - 109x + 30 = 0$ $x = -5, x = -2, x = \frac{1}{3}, x = 3$