6E Integrating Hyperbolics

1. Find

a)

$$\int \cosh(4x - 1)\ dx$$

b)

$$\int \left(\frac{2 + 5x}{\sqrt{x^2 + 1}}\right) dx$$

c)

$$\int cosh^5 2x \; sinh2x \; dx$$

d)

$$\int tanhx \; dx$$

e)

$$\int cosh^2 3x \; dx$$

f)

$$\int sinh^3 x\, dx$$

g)

$$\int e^{2x} \sinh x \;\, dx$$

2. By using an appropriate substitution, find:

$$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx\,,\ x > a$$

Formula book reference:

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\operatorname{arcosh} x = \ln\{x + \sqrt{x^2 - 1}\} \quad (x \geqslant 1)$

$\operatorname{arsinh} x = \ln\{x + \sqrt{x^2 + 1}\}$

$\operatorname{artanh} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

Integration (+ constant; $a > 0$ where relevant)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\arcsin\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\operatorname{arcosh}\left(\frac{x}{a}\right),\ \ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\operatorname{arsinh}\left(\frac{x}{a}\right),\ \ln\{x + \sqrt{x^2 + a^2}\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert = \frac{1}{a}\operatorname{artanh}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\frac{1}{1-x^2}$

3. Show that

a)

$$\int_5^8 \frac{1}{\sqrt{x^2 - 16}}\, dx = ln\left(\frac{2+\sqrt{3}}{2}\right)$$

b)

$$\int \sqrt{1+x^2}\ dx = \frac{1}{2} arsinhx + \frac{1}{2} x\sqrt{1+x^2} + c$$

4. By using a hyperbolic substitution, evaluate:

$$\int_0^6 \frac{x^3}{\sqrt{x^2+9}}\,dx$$

5.

$$\int \frac{1}{\sqrt{12x + 2x^2}}\, dx$$

6. Use the substitution

$$x = \frac{1}{2}(3 + 4\cosh u)$$

to find:

$$\int \frac{1}{\sqrt{4x^2 - 12x - 7}}\, dx$$