Standard Integrals

Same as non-hyperbolic version?	Integral	
✗	$\int \sinh x \; dx = \cosh x + C$	
✓	$\int \cosh x \; dx = \sinh x + C$	
✓	$\int \operatorname{sech}^2 x \; dx = \tanh x + C$	
✓	$\int \operatorname{cosech}^2 x \; dx = -\coth x + C$	
✗	$\int \operatorname{sech} x \tanh x \; dx = -\operatorname{sech} x + C$	
✓	$\int \operatorname{cosech} x \coth x \, dx = -\operatorname{cosech} x + C$	
	$\int \frac{1}{\sqrt{1-x^2}} \; dx = \arcsin x + C, \qquad \lvert x \rvert < 1$	
	$\int \frac{1}{1+x^2} \; dx = \arctan x + C$	
	$\int \frac{1}{\sqrt{1+x^2}} \; dx = \operatorname{arcsinh} x + C$	
	$\int \frac{1}{\sqrt{x^2-1}} \; dx = \operatorname{arccosh} x + C, \qquad x > 1$	

Not in this chapter but worth briefly mentioning. (sech², cosech², sech tanh, cosech coth)

Not in formula booklet. (cosech², sech tanh, cosech coth)

Was covered in Chapter 3. (arcsin, arctan)

Standard Integral Examples

$$\int \cosh(4x-1)\, dx =$$

$$\int sinh\left(\frac{2}{3}x\right)\; dx =$$

$$\int \frac{3}{\sqrt{1+x^2}} dx =$$

$$\int \frac{4}{\sqrt{x^2-1}} dx =$$

$$\int sinh\,(3x)\; dx =$$

$$\int \frac{10}{\sqrt{x^2-1}} dx =$$

$$\int \frac{2}{\sqrt{1+x^2}} dx =$$

Not Quite so Standard Examples

1. $\int \frac{2+5x}{\sqrt{x^2+1}}\ dx$

2. $\int \cosh^5 2x \sinh 2x\ \ dx$

3. $\int \tanh x\ \ dx$

Using Identities

1. $\int \cosh^2 3x \; dx$

2. $\int \sinh^3 x \; dx$

What now??

Sometimes there are techniques which work on non-hyperbolic trig functions but doesn't work on hyperbolic ones. Just first replace any hyperbolic functions with their definition.

Examples

1. Find $\int e^{2x} \sinh x \ dx$

2. Find $\int \operatorname{sech} x \ dx$

Dealing with $1/\sqrt{a^2+x^2}$, $1/\sqrt{x^2-a^2}$, ….

We can use substitution to deal with this style of integration.

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \sinh^2 u = \cosh^2 u$$

- Consider $\int \frac{1}{\sqrt{a^2+x^2}}\,dx$. What substitution might we use?

- Consider $\int \frac{1}{\sqrt{a^2+x^2}}\,dx$. What substitution might we use?

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx =$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx =$$

Example

1. Show that $\int \frac{1}{\sqrt{x^2-a^2}} dx = arcosh\left(\frac{x}{a}\right) + c$

2. Show that $\int_5^8 \frac{1}{\sqrt{x^2-16}} dx = \ln\left(\frac{2+\sqrt{3}}{2}\right)$

3. Show that $\int \sqrt{1+x^2}\,dx = \frac{1}{2} arsinh\, x + \frac{1}{2} x\sqrt{1+x^2} + C$.

Test Your Understanding

[June 2013 Q2]

(*a*) Find

$$\int \frac{1}{\sqrt{(4x^2+9)}}\, dx$$

(2)

(*b*) Use your answer to part (*a*) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2+9)}}\, dx$$

giving your answer in the form $k \ln(a + b\sqrt{5})$, where *a* and *b* are integers and *k* is a constant.

(3)

2)Using a hyperbolic substitution, evaluate $\int_0^6 \frac{x^3}{\sqrt{x^2+9}}\, dx$

Integrating by Completing the Square

By completing the square, we can then use one of the standard results.

Examples

1. Determine $\int \frac{1}{x^2-8x+8}\,dx$

2. Determine $\int \frac{1}{\sqrt{12x+2x^2}}\,dx$

Test Your Understanding

[June 2014(R) Q2]

$$9x^2 + 6x + 5 \equiv a(x + b)^2 + c$$

(*a*) Find the values of the constants a, b and c. **(3)**

Hence, or otherwise, find

(*b*) $\displaystyle\int \frac{1}{9x^2 + 6x + 5}\,dx$ **(2)**

(*c*) $\displaystyle\int \frac{1}{\sqrt{9x^2 + 6x + 5}}\,dx$ **(2)**

Ex 6E and Mixed Ex 6