

## 6.4) Trigonometric identities

## Worked example

Using  $\sin^2 x + \cos^2 x \equiv 1$ , prove that:  
 $1 + \tan^2 x = \sec^2 x$

## Your turn

Using  $\sin^2 x + \cos^2 x \equiv 1$ , prove that:  
 $1 + \cot^2 x = \operatorname{cosec}^2 x$

**Proof**

## Worked example

Prove that:

$$\operatorname{cosec}^2 \theta - \sin^2 \theta \equiv \cos^2 \theta (1 + \operatorname{cosec}^2 \theta)$$

## Your turn

Prove that:

$$\sec^2 \theta - \cos^2 \theta \equiv \sin^2 \theta (1 + \sec^2 \theta)$$

**Proof**

## Worked example

Prove that:

$$\sec^4 \theta - \tan^4 \theta = \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}$$

## Your turn

Prove that:

$$\operatorname{cosec}^4 \theta - \cot^4 \theta = \frac{1 + \cos^2 \theta}{1 - \cos^2 \theta}$$

**Proof**

## Worked example

Solve in the interval  $0^\circ \leq \theta \leq 360^\circ$ :

$$5 \sec^2 \theta - 1 = 9 \tan \theta$$

## Your turn

Solve in the interval  $0^\circ \leq \theta \leq 360^\circ$ :

$$4 \sec^2 \theta - 9 = \tan \theta$$

$$\theta = 51.3^\circ, 135.0^\circ, 231.3^\circ, 315.0^\circ \text{ (1 dp)}$$

## Worked example

Solve in the interval  $0^\circ \leq \theta \leq 360^\circ$ :

$$5 \operatorname{cosec}^2 \theta - 1 = 9 \cot \theta$$

## Your turn

Solve in the interval  $0^\circ \leq \theta \leq 360^\circ$ :

$$4 \operatorname{cosec}^2 \theta - 9 = \cot \theta$$

$$\theta = 38.7^\circ, 135.0^\circ, 218.7^\circ, 315.0^\circ \text{ (1 dp)}$$

## Worked example

Given that  $x = \operatorname{cosec} \theta + \cot \theta$ , express in its simplest form:

$$x^2 + \frac{1}{x^2} + 2$$

## Your turn

Given that  $x = \sec \theta + \tan \theta$ , express in its simplest form:

$$x^2 + \frac{1}{x^2} + 2$$
$$4 \sec^2 \theta$$