

6D Differentiating Hyperbolics

1. Show that $\frac{d}{dx}(\sinh x) = \cosh x$

2. Show that $\frac{d}{dx}(\cosh x) = \sinh x$

3. Show that $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

4. Differentiate $\cosh 3x$ with respect to x

5. Differentiate $x^2 \cosh 4x$ with respect to x

6. Given that:

$$y = A \cosh 3x + B \sinh 3x$$

Where A and B are constants, prove that $\frac{d^2y}{dx^2} = 9y$

7. Show that $\frac{d}{dx}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2-1}}$, $x > 1$

8. Given $y = x \operatorname{arcosh} x$, find $\frac{dy}{dx}$

9. Given $y = (\operatorname{arcosh} x)^2$, prove that:

$$(x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 4y$$

10.

a) Show that $\frac{d}{dx} (\operatorname{arsinh} x) = \frac{1}{\sqrt{1+x^2}}$

b) Find the first two non-zero terms in the series expansion of $\operatorname{arsinh} x$

c) The general term for the series expansion of $\operatorname{arsinh} x$ is given by:

$$\operatorname{arsinh} x = \sum_{r=0}^{\infty} \left(\frac{(-1)^n (2n)!}{2^{2n} (n!)^2} \right) \frac{x^{2n+1}}{2n+1}$$

Find, in its simplest terms, the third term in the sequence

d) Use your approximation, up to and including the term in x^5 , to find an approximation for $\operatorname{arsinh} 0.5$

e) Calculate the percentage error by using this approximation