Differentiating hyperbolic functions

$\frac{d}{dx}(\sinh x) = \cosh x$	$\frac{d}{dx}(\tanh x) = \text{sech}^2\, x$
$\frac{d}{dx}(\cosh x) = \sinh x$	$\frac{d}{dx}(\coth x) = -cosech^2 x$

Example

Prove that $\frac{d}{dx}(\sinh x) = \cosh x$

Test Your Understanding

[June 2014 (R) Q3] 6. The curve C has equation

$$y = \frac{1}{2}\ln(\coth x), \qquad x > 0$$

(*a*) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\text{cosech}\, 2x$$ **(3)**

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\operatorname{arsinh} x) = \frac{1}{\sqrt{x^2+1}}$$
$$\frac{d}{dx}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\operatorname{artanh} x) = \frac{1}{1-x^2}$$

Proof of $\frac{d}{dx}(\operatorname{arsinh} x) = \frac{1}{\sqrt{x^2+1}}$

Examples

1. Find $\frac{d}{dx}(artanh\ 3x)$

2. Given that $y = (arcosh\, x)^2$ prove that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y$

Test Your Understanding

[June 2009 Q4] Given that $y = \text{arsinh}(\sqrt{x}),\quad x > 0,$

(*a*) find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer as a simplified fraction. **(3)**

[June 2010 Q5] Given that $y = (\text{arcosh } 3x)^2$, where $3x > 1$, show that

(*a*) $(9x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 36y$, **(5)**

(*b*) $(9x^2 - 1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18.$ **(4)**

Using Maclaurin expansions for approximations

Textbook Example

(a) Show that $\frac{d}{dx}(arsinh\, x) = \frac{1}{\sqrt{1+x^2}}$ **[We did this earlier]**

(b) Find the first two non-zero terms of the series expansion of $arsinh\, x$.

The general form for the series expansion of $arsinh\, x$ is given by

$$arsinh\, x = \sum_{r=0}^{\infty} \left(\frac{(-1)^n (2n)!}{2^{2n}(n!)^2} \right) \frac{x^{2n+1}}{2n+1}$$

(c) Find, in simplest terms, the coefficient of x^5.

(d) Use your approximation up to and including the term in x^5 to find an approximate value for $arsinh\, 0.5$.

(e) Calculate the percentage error in using this approximation.

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