Hyperbolic Identities

Use the definitions of $sinh$ and $cosh$ to find:

$\mathbf{\cosh^2 x - \sinh^2 x =}$

$\boldsymbol{1 - tanh^2 x =}$

$\boldsymbol{coth^2 x - 1 =}$

Also:

$$\boldsymbol{sinh\,(A \pm B) = sinh\,A\,cosh\,B \pm cosh\,A\,sinh\,B}$$

$$\boldsymbol{cosh\,(A \pm B) = cosh\,A\,cosh\,B \pm sinh\,A\,sinh\,B}$$

Hence:

$$\boldsymbol{tanh\,(A \pm B) =}$$

Osborn's Rule

We can get these identities from the normal sin/cos ones using Osborn's rule:

Osborn's Rule:

1. Replace $sin \rightarrow sinh$ and $cos \rightarrow cosh$
2. **Negate** any explicit or implied **product of two sines**.

$$\sin A \sin B =$$

$$\tan^2 A \quad =$$

$$cos2A = 2\cos^2 A - 1 \quad \rightarrow$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad \rightarrow$$

Solving Equations

To solve equations either use hyperbolic identities or basic definitions of hyperbolic functions.

Examples

1. Solve for all real x, $6\sinh x - 2\cosh x = 7$

2. Solve for all real x, $2\cosh^2 x - 5sinh\, x = 5$

Recap: If $\cos x = \frac{3}{5}$, find $\sin x$

3. If $\sinh x = \frac{3}{4}$, find the exact value of:
   a) $\cosh x$
   b) $\tanh x$
   c) $\sinh 2x$

Test Your Understanding

1.

[FP3 June 2009 Q1] Solve the equation

$$7 \text{ sech } x - \tanh x = 5$$

Give your answers in the form $\ln a$, where a is a rational number. **(5)**

2.

[FP3 June 2014 (I) Q3] Using the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials,

(*a*) prove that

$$\cosh^2 x - \sinh^2 x \equiv 1$$

(2)

(*b*) find algebraically the exact solutions of the equation

$$2 \sinh x + 7 \cosh x = 9$$

giving your answers as natural logarithms. **(5)**

3.

[FP3 June 2011 Q5]

(*b*) Solve the equation $3 \sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2} \ln k$, where k is an integer.

(5)

Ex 6C, pg 128