6.3) Cumulative probabilities

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp,

a) $P(X = 3)$
b) $P(X \leq 5)$
c) $P(X < 5)$
d) $P(X \geq 7)$
e) $P(X > 7)$
f) $P(4 < X < 9)$
g) $P(4 \leq X \leq 9)$
h) $P(4 \leq X < 9)$
i) $P(4 < X \leq 9)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp,

a) $P(X = 4)$
b) $P(X \leq 6)$
c) $P(X < 6)$
d) $P(X \geq 8)$
e) $P(X > 8)$
f) $P(5 < X < 10)$
g) $P(5 \leq X \leq 10)$
h) $P(5 \leq X < 10)$
i) $P(5 < X \leq 10)$

a) 0.0345
b) 0.2500
c) 0.1256
d) 0.5841
e) 0.4044
f) 0.6297
g) 0.8215
h) 0.7044
i) 0.7469

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X = 3)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X = 4)$

0.0345

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X \leq 5)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X \leq 6)$

0.2500

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp,
$P(X \text{ is at most } 5)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp,
$P(X \text{ is at most } 6)$

0.2500

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X < 5)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X < 6)$

0.1256

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X \geq 7)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X \geq 8)$

0.5841

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp,
$P(X \text{ is at least } 7)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp,
$P(X \text{ is at least } 8)$

0.5841

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X > 7)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X > 8)$

0.4044

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 < X < 9)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 < X < 10)$

0.6297

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 \leq X \leq 9)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 \leq X \leq 10)$

0.8215

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp,
$P(4 \leq X < 9)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp,
$P(5 \leq X < 10)$

0.7044

Worked example

Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 < X \leq 9)$

Your turn

Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 < X \leq 10)$

0.7469

Worked example

A spinner is designed so that probability it lands on red is 0.2.
Jane decides to use this spinner for a class competition.
She wants the probability of winning a prize to be less than 0.03.
Each member of the class will have 15 spins and the number of reds will be recorded.
Find how many reds are needed to win the prize.

Your turn

A spinner is designed so that probability it lands on red is 0.3.
Jane decides to use this spinner for a class competition.
She wants the probability of winning a prize to be less than 0.05.
Each member of the class will have 12 spins and the number of reds will be recorded.
Find how many reds are needed to win the prize.

7 or more

Worked example

At a university, students have 10 exams at the end of the year.
All students pass each individual exam with probability 0.55.
Students are only allowed to continue into the next year if they pass some minimum of exams out of the 10.
What do the university administrators need to set this minimum number such that the probability of continuing to next year is at least 80%?

Your turn

At a university, students have 20 exams at the end of the year.
All students pass each individual exam with probability 0.45.
Students are only allowed to continue into the next year if they pass some minimum of exams out of the 20.
What do the university administrators need to set this minimum number such that the probability of continuing to next year is at least 90%?

6

Worked example

The random variable $X \sim B(40, 0.3)$. Find:

a) The largest value of p such that $P(X \leq p) < 0.05$

b) The largest value of r such that $P(X < r) < 0.1$

c) The smallest value of s such that $P(X \geq s) < 0.15$

d) The smallest value of t such that $P(X > t) < 0.2$

Your turn

The random variable $X \sim B(30, 0.4)$. Find:

a) The largest value of p such that $P(X \leq p) < 0.2$

b) The largest value of r such that $P(X < r) < 0.15$

c) The smallest value of s such that $P(X \geq s) < 0.1$

d) The smallest value of t such that $P(X > t) < 0.05$

a) $p = 9$

b) $r = 9$

c) $s = 16$

d) $t = 16$

Worked example

The random variable $X \sim B(40, 0.3)$. Find the largest value of p such that $P(X \leq p) < 0.05$

Your turn

The random variable $X \sim B(30, 0.4)$. Find the largest value of p such that $P(X \leq p) < 0.2$

$p = 9$

Worked example

The random variable $X \sim B(40, 0.3)$. Find the largest value of r such that $P(X < r) < 0.1$

Your turn

The random variable $X \sim B(30, 0.4)$. Find the largest value of r such that $P(X < r) < 0.15$

$r = 9$

Worked example

The random variable $X \sim B(40, 0.3)$. Find the smallest value of s such that $P(X \geq s) < 0.15$

Your turn

The random variable $X \sim B(30, 0.4)$. Find the smallest value of s such that $P(X \geq s) < 0.1$

$s = 16$

Worked example

The random variable $X \sim B(40, 0.3)$. Find the smallest value of t such that $P(X > t) < 0.2$

Your turn

The random variable $X \sim B(30, 0.4)$. Find the smallest value of t such that $P(X > t) < 0.05$

$t = 16$

Worked example

Each day a person plays 10 games of chess. The probability that they win each game is 0.7.
They consider it a successful day if they win at least 8 games.
Calculate the probability that in a seven-day week, they have at least five successful days.

Your turn

Each day a person plays 20 games of chess. The probability that they win each game is 0.6.
They consider it a successful day if they win at least 13 games.
Calculate the probability that in January they have at least sixteen successful days.

0.1708 (4 dp)