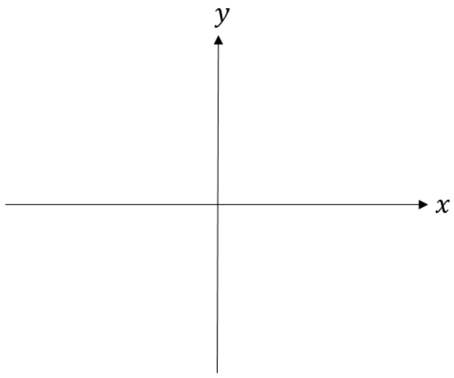
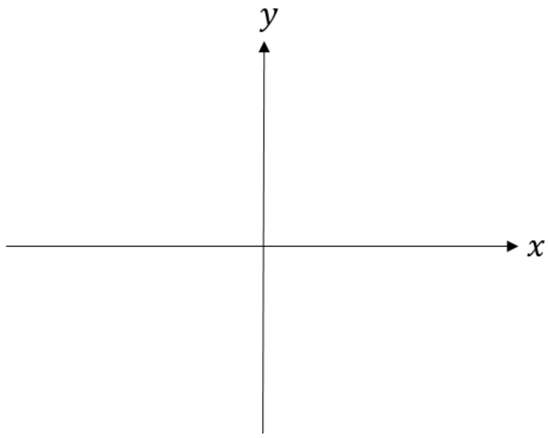
Inverse Hyperbolic Functions

Each hyperbolic function has an inverse.

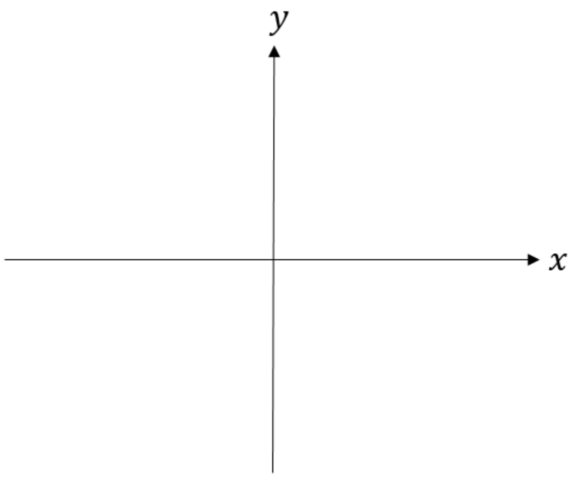
$y = \boldsymbol{arsinh\ x}$,



$y = \boldsymbol{arcosh\ x}$,



$y = \boldsymbol{artanh\ x}$



$y = \boldsymbol{arsech\ x}$,

$y = \boldsymbol{arcosech\ x}$,

$y = \boldsymbol{arcoth\ x}$

Expressing Inverse Hyperbolic Functions in terms of ln

Given that hyperbolic functions can be written in terms of e inverse hyperbolic can be expressed in terms of ln.

Example:

Prove that $arsinh\, x = \ln\left(x + \sqrt{x^2+1}\right)$

Test Your Understanding

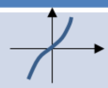
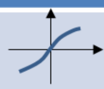
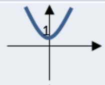
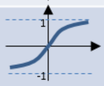
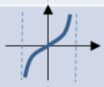
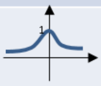
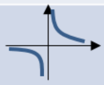
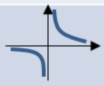
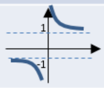
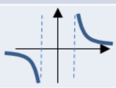
Prove that $arcosh\, x = \ln\left(x + \sqrt{x^2-1}\right),\ x \geq 1$

Summary so far:

$$arsinh\,x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$arcosh\,x = \ln\left(x + \sqrt{x^2-1}\right), \quad x \geq 1$$

$$artanh\,x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad |x| < 1$$

Hyperbolic	Domain	Sketch	Inverse Hyperbolic	Domain	Sketch
$y = \sinh x$	$x \in \mathbb{R}$		$y = arsinh\,x$	$x \in \mathbb{R}$	
$y = \cosh x$	$x \geq 0$		$y = arcosh\,x$	$x \geq 1$	
$y = \tanh x$	$x \in \mathbb{R}$		$y = artanh\,x$	$\|x\| < 1$	
$y = \text{sech}\,x$	$x \geq 0$		$y = arsech\,x$	$0 < x \leq 1$	
$y = cosech\,x$	$x \neq 0$		$y = arcosech\,x$	$x \neq 0$	
$y = \coth x$	$x \neq 0$		$y = \text{arcoth}\,x$	$\|x\| > 1$	

Ex 6B pg 125