CP2 Chapter 6

Hyperbolic Functions

Course Structure

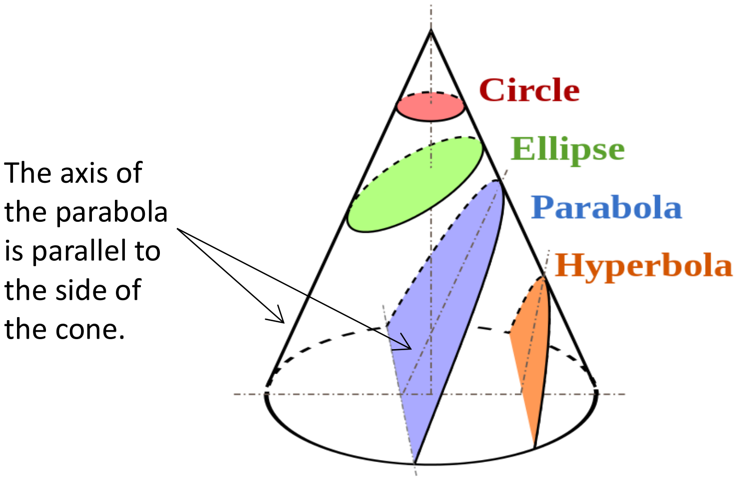
1. Definition of hyperbolic functions and their sketches.
2. Inverse hyperbolic functions.
3. Hyperbolic Identities and Solving Equations
4. Differentiation
5. Integration

8 **Hyperbolic functions**	8.1	Understand the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and be able to sketch their graphs.	For example, $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$
	8.2	Differentiate and integrate hyperbolic functions.	For example, differentiate $\tanh 3x$, $x \sinh^2 x$, $\frac{\cosh 2x}{\sqrt{(x+1)}}$
8 **Hyperbolic functions** *continued*	8.3	Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.	$\text{arsinh} x = \ln\left[x + \sqrt{x^2+1}\right]$ $\text{arcosh} x = \ln\left[x + \sqrt{x^2-1}\right],\ x \geqslant 1$ $\text{artanh} x = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right],\ -1 < x < 1$
	8.4	Derive and use the logarithmic forms of the inverse hyperbolic functions.	
	8.5	Integrate functions of the form $\left(x^2+a^2\right)^{-\frac{1}{2}}$ and $\left(x^2-a^2\right)^{-\frac{1}{2}}$ and be able to choose substitutions to integrate associated functions.	

<u>Conic Sections</u>

In mathematics there are a number of different **families of curves**. Each of these have different properties and their equations have different forms.

It is possible to obtain these different types of curves by **slicing a cone**, hence "conic sections".

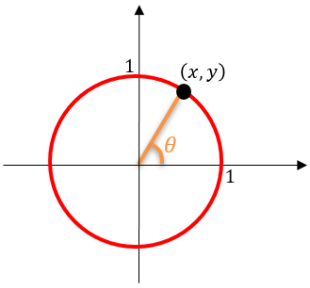


For interest:

Comparing circles and hyperbolas

(Don't make notes on this slide) You will cover Hyperbolas in FP1, but this will give some context for the eponymously named 'hyperbolic functions' that we will explore in this chapter.

Circles



The 'simplest' circle is a unit circle centred at the origin.
Cartesian equation:
$$x^2 + y^2 = 1$$

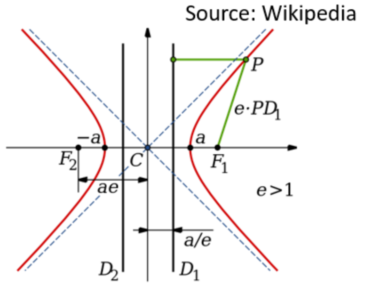
Parametric eqns (in terms of θ):
$$x = \cos\theta$$
$$y = \sin\theta$$

similar

similar

Hyperbolas

Source: Wikipedia



The equivalent hyperbola (which crosses x-axis at $(1, 0)$ and $(-1, 0)$)
Cartesian equation:
$$x^2 - y^2 = 1$$

Parametric equations:
$$x = \cosh\theta$$
$$y = \sinh\theta$$

What's the point of hyperbolic functions?

Hyperbolic functions often result from differential equations (e.g. in mechanics), and we'll see later in this module how we can use these functions in calculus.

For example, we can consider forces acting on each point on a hanging piece of string.

Solving the relevant differential equations, we end up with coshx.

Equations for Hyperbolic Functions

Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad x \in \mathbb{R}$$

Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad x \in \mathbb{R}$$

Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x} - 1}{e^{2x} + 1} \qquad x \in \mathbb{R}$$

Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} \qquad x \in \mathbb{R}$$

Hyperbolic cosecant:

$$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \qquad x \in \mathbb{R},\ x \neq 0$$

Hyperbolic cotangent:

$$\coth x = \frac{1}{\tanh x} = \frac{e^{2x} + 1}{e^{2x} - 1} \qquad x \in \mathbb{R},\ x \neq 0$$

Examples:

1. Calculate (using both your $sinh$ button and using the formula)

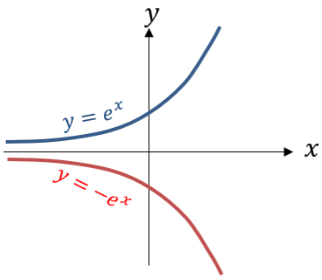
$$\mathbf{\sinh 3 = 10.02}$$

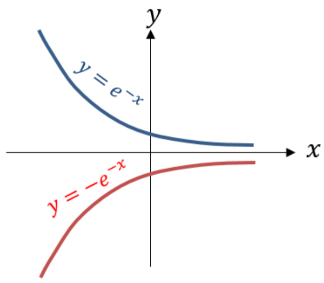
2. Write in terms of e: $\mathbf{cosech\,3}$

3. Find the exact value of: $\boldsymbol{tanh\,(ln\,4)}$

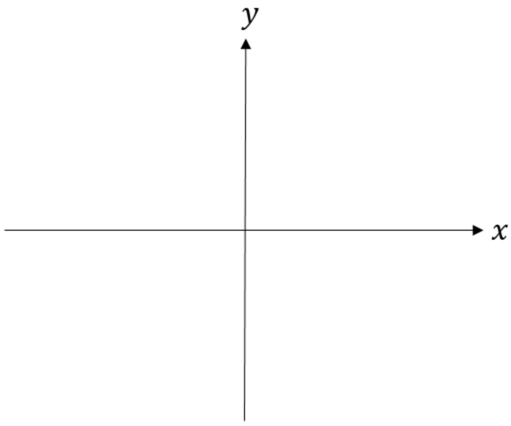
4. Solve $sinh\,x = 5$

Sketching Hyperbolic Functions

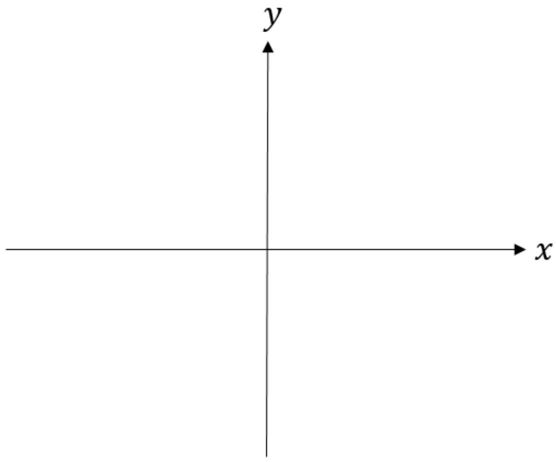




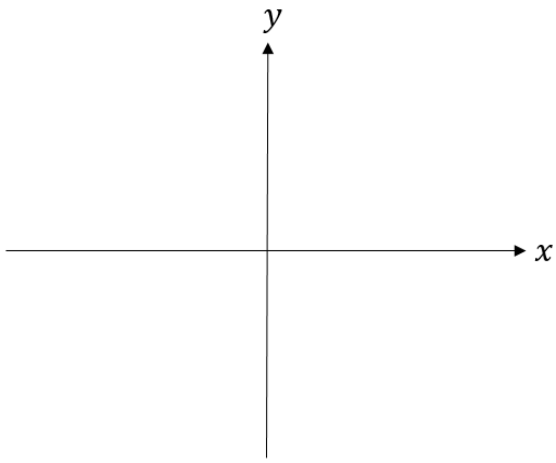
- $y = \sinh x$ is the average of e^x and $-e^{-x}$:



- $y = \cosh x$ is the average of e^x and e^{-x}:

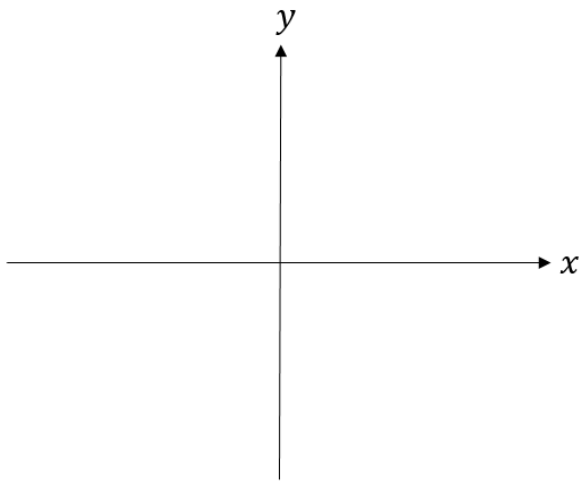


- $\tanh x = \dfrac{\sinh x}{\cosh x}$



Test Your Understanding

Sketch the graph of $y = \text{sech}\, x$



[FP3 June 2011 Q5] The curve C_1 has equation $y = 3 \sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

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