

6.1) Probability distributions

Worked example

Let X = number of tails when a fair coin is tossed 4 times.

Write a list of all the possible outcomes.

Your turn

Let X = number of tails when a fair coin is tossed 3 times.

Write a list of all the possible outcomes.

HHH

THH

HTH

HHT

TTH

THT

HTT

TTT

Worked example

Let X = number of tails when a fair coin is tossed 4 times.

Describe the probability distribution of X :

- Using a table

Your turn

Let X = number of tails when a fair coin is tossed 3 times.

Describe the probability distribution of X :

- Using a table

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Worked example

Let X = number of tails when a fair coin is tossed 4 times.

Describe the probability distribution of X :

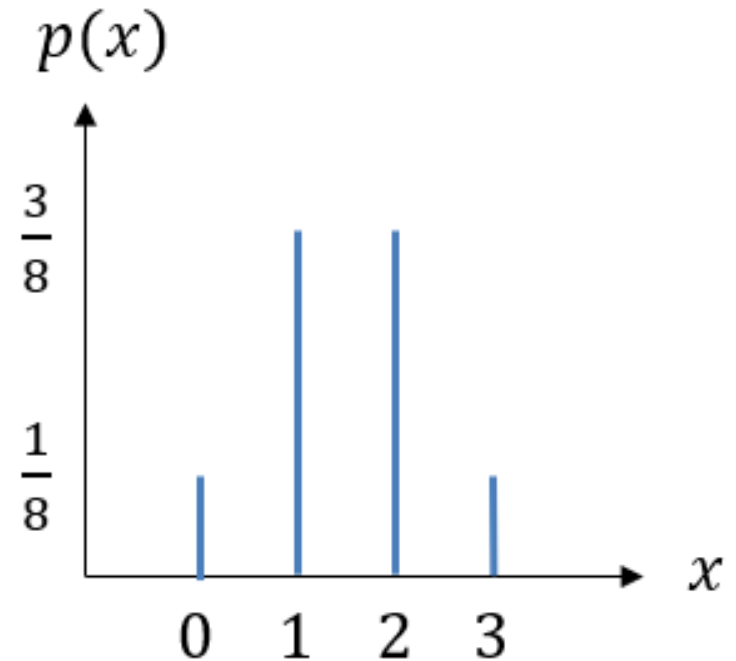
- Using a diagram

Your turn

Let X = number of tails when a fair coin is tossed 3 times.

Describe the probability distribution of X :

- Using a diagram



Worked example

Let X = number of tails when a fair coin is tossed 4 times.

Describe the probability distribution of X :

- As a probability mass function

Your turn

Let X = number of tails when a fair coin is tossed 3 times.

Describe the probability distribution of X :

- As a probability mass function

$$P(X = x) = \begin{cases} \frac{1}{8} & x = 0, 3 \\ \frac{3}{8} & x = 1, 2 \\ 0 & \textit{otherwise} \end{cases}$$

Worked example

A biased six-sided dice with faces numbered 1, 2, 3, 4, 5 and 6 is rolled. The number on the bottom-most face is modelled as a random variable X .

Given that $P(X = x) = \frac{k}{x}$,

- Find the value of k
- Give the probability distribution of X in table form
- Find the probability that:
 - $X \geq 2$
 - $1 \leq X < 4$
 - $X < 1$
 - $2X + 1 > 11$

Your turn

A biased four-sided dice with faces numbered 1, 2, 3 and 4 is rolled.. The number on the bottom-most face is modelled as a random variable X .

Given that $P(X = x) = \frac{k}{x}$,

- Find the value of k
- Give the probability distribution of X in table form
- Find the probability that:
 - $X > 2$
 - $1 \leq X < 4$
 - $X \leq 4$
 - $3X - 5 < 0$

a) $k = \frac{12}{25}$

b)

x	1	2	3	4
$P(X = x)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

c)

i) $\frac{7}{25}$

ii) $\frac{22}{25}$

iii) 1

iv) $\frac{12}{25}$

Worked example

The random variable X has a probability function

$$P(X = x) = \frac{k}{x^3}, \quad x = 1, 2, 3, 4$$

Find the value of k

Your turn

The random variable X has a probability function

$$P(X = x) = \frac{k}{x^2}, \quad x = 1, 2, 3, 5$$

Find the value of k

$$k = \frac{900}{1261}$$

Worked example

The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3 \\ k(x - 2) & x = 2, 4 \end{cases}$$

- a) Find the value of k
- b) Find $P(X > 1)$

Your turn

The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 2 \\ k(x - 3) & x = 3, 4 \end{cases}$$

- a) Find the value of k
- b) Find $P(X < 4)$

a) $k = \frac{1}{4}$

b) $\frac{3}{4}$

Worked example

The random variable X has a probability function

$$P(X = x) = \begin{cases} k(2 - x)^2 & x = -2, -1, 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k

Your turn

The random variable X has a probability function

$$P(X = x) = \begin{cases} k(1 - x)^2 & x = -1, 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k

$$k = \frac{1}{6}$$

Worked example

A spinner has six equally-sized sections.
Four contain the letter G. 2 contain the letter Y.
The spinner is spun until it lands on Y or has been spun five times in total.
Find the probability distribution of the random variable S , the number of times the spinner is spun.

Your turn

A spinner has five equally-sized sections.
Three contain the letter B. 2 contain the letter R.
The spinner is spun until it lands on R or has been spun four times in total.
Find the probability distribution of the random variable S , the number of times the spinner is spun.

s	1	2	3	4
$P(S = s)$	$\frac{2}{5}$	$\frac{6}{25}$	$\frac{18}{125}$	$\frac{27}{125}$

Worked example

The random variable X can take any integer value from 1 to 30. Given that X has a discrete uniform distribution, find:

- a) $P(X = 5)$
- b) $P(X \geq 20)$
- c) $P(12 < X < 21)$

Your turn

The random variable X can take any integer value from 1 to 40. Given that X has a discrete uniform distribution, find:

- a) $P(X = 3)$
- b) $P(X \geq 21)$
- c) $P(13 < X < 31)$

- a) $\frac{1}{40}$
- b) $\frac{1}{2}$
- c) $\frac{17}{40}$

Worked example

A discrete random variable has a probability distribution as shown in the table. Find the value of a

x	0	1	2	3
$P(X = x)$	a	$a - \frac{1}{4}$	$a + \frac{1}{3}$	$3a$

Your turn

A discrete random variable has a probability distribution as shown in the table. Find the value of a

x	1	2	3	4
$P(X = x)$	$2a$	$a - \frac{1}{3}$	$a + \frac{1}{4}$	$5a$

$$a = \frac{13}{108}$$